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Abstract

Stability results for an open economy DSGE adapted to an emerging market (SOEME) with a dualistic structure have the same structure as in the original model, but those derived for the simulated version turn out to impose no restriction on the coefficient of inflation, but rather a threshold on the coefficient of the output gap. Other rigidities, lags and some degree of backward looking behavior in the simulated SOEME model arising from its calibration to an emerging market, may be helping provide a nominal anchor. Estimation of a Taylor rule for India, simulations in the SOEME model itself and a variant with government debt, confirm the analytical result. Implications are, first, optimization can be as effective as following a monetary policy rule. Second, knowledge of the specific rigidities in an economy can give useful inputs for the design of policy—their effect on stability should be more carefully researched.

Keywords:

DSGE, emerging economy, rigidities, stability, optimization, Taylor rule

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1. Introduction

We examine basic stability issues in a dynamic stochastic general equilibrium model for a small open economy (SOE), adapted to an emerging market with two classes (high and low productivity) of consumer-workers[1] (SOEME).

Models with forward-looking behavior can easily be unstable. Sargent and Wallace (1975) demonstrated that with rational expectations prices are indeterminate under an interest rate instrument. But McCallum (1981) later showed indeterminacy only occurs if the Central Bank (CB) places no weight on prices in its response. The basic point is some nominal anchor is required to fix the price or inflation level in an economy. The CB response to inflation, or its targeting of nominal money stock, provides such an anchor, but many other rigidities and lags may serve to anchor inflation (Friedman, 1990).

The NKE literature (Woodford, 2003) has a new justification for a large CB response to inflation. Apart from its contribution to stability, it prevents higher prices today being set in response to expected excess demand. If prices are sticky, setting higher prices today creates inflation persistence. Therefore the literature justifies a Taylor type monetary policy rule, with the CB's response to inflation, exceeding unity. Moreover, it is shown analytically, in the canonical forward-looking model, that such a response exceeding unity is necessary for stability. The implication is a monetary policy rule can perform better than optimization.

A monetary policy rule can also be justified as a credible commitment, preventing opportunistic behavior from the CB resulting in an inflation bias. If future policy affects the output gap and future output gaps affect inflation credible policy may reduce inflation with lower output cost. This argument holds even if the CB has no inflation bias, as in emerging democracies where inflation hurts the poor and loses votes (Goyal, 2007). But in a discretionary optimum also, even though the CB

[1] See Gali and Monacelli (2005), Goyal (2011a).

reoptimizes every period, it has no incentive to create surprise inflation, and the private sector recognizes this (Clarida et. al. 1999).

We derive the stability result for the canonical SOEME, and it turns out to have the same structure as the result for the SOE. The simulated SOEME model, however, has more lags and some degree of backward looking behavior arising from its calibration to an emerging market with many types of rigidities. The stability result derived for the simulated model turns out to have a very different structure. There is no restriction on the coefficient of inflation, but rather a threshold on the coefficient of the output gap. Different types of rigidities may be helping provide a nominal anchor.

The analytical result is explored through simulations in the SOEME model itself and in a variant that also adds an equation for the evolution of government debt. The debt model also demonstrates the contribution of rigidities in relaxing stability conditions.

The implication is consumer-welfare based optimization can be as effective as following a monetary policy rule. If a rule is followed, the response coefficients need not exceed unity. The practice of monetary policy in India is consistent with these results. Analytical solutions of responses to monetary policy shocks in the SOEME model with a monetary policy rule are directionally similar to simulated responses in the calibrated optimizing model. The calibrated CB's reaction function in the SOEME, and a Taylor-type monetary policy rule estimated with Indian data both show response coefficients much smaller than unity.

The result may apply more generally than just to emerging markets to the extent advanced economies also have many types of frictions.

The structure of the paper is as follows: Section 2 presents a basic SOEME; section 3 derives the implications of stability for a policy rule; section 4 discusses analytical and simulated responses to shocks; section 5 presents estimated monetary policy rule, before section 6 concludes. Derivations are in appendices.

2. A Small Open Emerging Market Model

A microfounded dynamic stochastic general equilibrium model for a small open emerging market derives the following aggregate demand (AD) (2) and supply curves AS (3) (Goyal 2011), subject to which the central bank (CB) minimizes a loss function (1), derived from consumers' welfare. The loss function is a weighted average of output, inflation and interest rate deviations from equilibrium values:

$$L = q_x x^2 + q_\pi \pi^2 + q_i i^2 \qquad (1)$$

The last is a smoothing term that prevents large changes in the policy rate, where i_t is the riskless nominal interest rate. The first is the output gap $x_t = y_t - \bar{y}_t$, and the second term, inflation, can be either domestic inflation, π_H, or consumer price inflation $\pi_t \equiv p_t - p_{t-1}$ (where $p_t \equiv \log P_t$). The CB minimizes (1) subject to the AD (2) and AS (3). The dynamic AD equation for the SOEME is:

$$x_t = E_t\{x_{t+1}\} - \frac{1}{\sigma_D}(i_t - E_t\{\pi_{H,t+1}\} - \overline{rr}_t) \qquad (2)$$

Where: $\overline{rr}_t = \rho - \sigma_D \Gamma(1-\rho_a)a_t - \sigma_D(1-\eta+\Phi)E_t\{\Delta c_{P,\ t+1}\} + \sigma_D(\Theta - \Psi)E_t\{\Delta y^*_{t+1}\}$

$$\Theta = \alpha(\varpi - \eta),\quad d = \frac{1}{\sigma_D + \varphi},\quad \Gamma = \frac{(1+\varphi)}{\sigma_D + \varphi}, \Psi = \eta(\sigma - \sigma_D)d,$$

$$\sigma_D = \frac{\sigma_R}{(\eta(1-\alpha) + \varpi\alpha)},\quad \Phi = d((1-\eta)(\sigma - \sigma_D)),\quad \varpi = \sigma_R + (1-\alpha)(\sigma_R - 1)$$

The dynamic AS is:

$$\pi_{H,t} = \gamma_f \beta E_t\{\pi_{H,t+1}\} + \kappa_D x_t + \gamma_b \pi_{H,t-1} \qquad \gamma_f + \gamma_b = 1 \qquad (3)$$

Lower case letters are logs of the respective variables. Since empirical estimations and the dominance of administered pricing in an emerging market (EM) suggests that past inflation affects current inflation, the AS (3) has a positive γ_b as the share of lagged and γ_f the share of forward-looking inflation. Table 1 explains the parameters and gives their calibrated values.

Table 1 about here

The model adapts the SOE model of Galí and Monacelli (2005) (GM) for an emerging market with two types R and P of consumer-workers. The R types, with population share η, 0< η <1, are able to smooth consumption in perfect capital markets. P types are assumed to be at a fixed subsistence wage, financed in part by transfers from R types. The labour supply elasticities of the P types are higher than the R types and the P type intertemporal elasticity of consumption is zero.

Marginal cost at its steady-state level, when prices are perfectly flexible, defines the natural output $\overline{y_t}$. But the world output level is the final steady-state for a SOEME. Low productivity, poor infrastructure and other distortions keep the natural output in an EM below world levels. Convergence to world levels is part of the process of development. Productivity shocks, a_t, can be more persistent in EMs that are in transition stages of upgrading technologies.

The steady-state natural interest rate, ρ, is defined as the equilibrium real rate, consistent with a zero or target rate of inflation, when prices are fully flexible in a SOE. It is also the time discount rate since $\rho \equiv \beta^{-1} - 1 = -\log \beta$ is where β is the discount factor. Shocks that change ρ open an output gap and affect inflation. The shocks in the term $\overline{rr}_t$ that enters the AD therefore lead to a deviation of the natural rate from its steady-state value. The deviation occurs due to real disturbances that change natural output; $\overline{rr}_t$ rises for any temporary demand shock and falls for any temporary supply shock. Optimal policy requires insulating the output gap from these shocks, so that the CB's interest rate instrument should move in step with the natural rate. Thus the CB would accommodate positive supply shocks that raise the natural output by lowering interest rates. It would offset positive demand shocks that raise output above its potential by raising interest rates. In an EM a reduction in c_P is an additional large shock requiring reduction in the policy rate, since it increases the distance from the world consumption level. The parameters of the other shock terms are also different. Goyal (2011a) systematically compares the differences in behaviour and outcomes for the SOE and SOEME. As η approaches unity the EM becomes developed and the SOEME collapses to the SOE.

In the next section we draw out the implications of structure and rigidities in the SOEME for stability. Forward-looking behaviour can imply instability and multiple equilibria with an interest rate rule. The NKE-SOE models have a policy rule that imposes stability. An equivalent rule can be derived for the SOEME, with a similar response coefficient for inflation that exceeds unity.

3. What stability implies for a policy rule

We demonstrate instability in the SOEME system comprising (2) and (3), and then show how an adequate policy response can impose stability. Full stabilization implies that $x_t = \pi_{H,t} = 0$, $y_t = \overline{y}_t$ and $r_t = \overline{rr}_t$. Substituting dynamic AD (2) in the AS[2] (3) to write the AS as a function of x_{t+1} the two equations become:

$$x_t = E_t\{x_{t+1}\} + \sigma_D^{-1} E_t\{\pi_{H,\,t+1}\} \tag{4}$$

$$\pi_{H,t} = \kappa_D E_t\{x_{t+1}\} + \left(\beta + \sigma_D^{-1}\kappa_D\right) E_t\{\pi_{H,\,t+1}\} \tag{5}$$

In matrix form they are:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = A_o \begin{bmatrix} E_t\{x_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} \quad \text{with } A_o = \begin{bmatrix} 1 & \sigma_D^{-1} \\ \kappa_D & \beta + \kappa\sigma_D^{-1} \end{bmatrix} \tag{6}$$

Since the determinant and trace of the coefficient matrix A_o are both greater than zero the system is unstable. Local indeterminacy is possible and sunspot fluctuations can occur.

But a major result in the literature is that an adequate policy response to inflation, that exceeds unity, can impose stability. The reason is that since sticky prices are set in a forward-looking manner, an early and robust policy response will prevent inflationary expectations entering this price setting and therefore lower inflation and the future costs of disinflating (Clarida et.al. 1999).

GM show analytically that a simple Taylor-type policy rule, with a coefficient on inflation that exceeds unity, is sufficient to ensure stability in the SOE. We derive the

[2] In order to simplify the derivations γ_b is taken as equal to 1.

equivalent stability condition for the SOEME. A simple policy rule whereby the interest rate is raised if there is domestic inflation or if the output gap is positive is:

$$i_t = \overline{rr}_t + \phi_\pi \pi_{H,t} + \phi_x x_t \tag{7}$$

Substituting for i_t minus its equilibrium value from the policy rule into (2), transforming π_{Ht} into π_t, and substituting for π_t, then substituting for x_t, with i substituted in it, in (3), we get:

$$(\sigma_D + \phi_x + \phi_\pi \kappa_D) x_t = \sigma_D E_t\{x_{t+1}\} + (1 - \phi_\pi \beta) - \phi_\pi \beta E_t\{\pi_{t+1}\} \tag{8}$$

$$(\sigma_D + \phi_x + \phi_\pi \kappa_D) \pi_t = \kappa_D \sigma_D E_t\{x_{t+1}\} + (\kappa_D + \beta(\sigma_D + \phi_x)) E_t\{\pi_{t+1}\} \tag{9}$$

The AD and AS (2) and (3) transformed to (8) and (9), as required for stability analysis, and written in matrix form are:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t\{x_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} \tag{10}$$

where

$$A_T = \Omega \begin{bmatrix} \sigma_D & 1 - \beta\phi_\pi \\ \sigma_D \kappa_D & \kappa_D + \beta(\sigma_D + \phi_x) \end{bmatrix} \text{and } \Omega = \frac{1}{\sigma_D + \phi_x + \kappa_D \phi_\Pi}$$

The stability condition[3] for a unique non-explosive solution is[4] $\kappa_D(\phi_\pi - 1) + (1 - \beta)\phi_x > 0$. A policy response to inflation that exceeds unity is sufficient to ensure stability. The result for the SOE in GM is the same, only the coefficient values are different. GM's κ becomes κ_D in the SOEME; $\overline{rr}_t$ is also different.

But a slightly modified version of the SOEME model turned out to be stable under optimizing simulations conducted with the parameters as given in Table 1.

[3] The stability condition for a two equation difference system is determinant A > 0, and determinant A+ trace A>-1 when the system is written in the form $z_t = E(z_{t+1}) + \ldots\ldots\ldots.$ (see Woodford, 2003).

$$x_{t+1} = \left(1 + \frac{\lambda}{\gamma_f \beta}\left(1 + \frac{\phi}{\sigma_D}\right)\right)x_t + \frac{1}{\sigma_D}\left(0.2\, r_{t-1} + 0.8 r_t + \frac{\pi_{H,t}}{\gamma_f \beta} + \frac{\gamma_b}{\gamma_f \beta}\pi_{H,t-1} - \overline{rr_t}\right) \quad (11)$$

$$\pi_{H,t+1} = \frac{1}{\gamma_f \beta}\pi_{H,t} - \frac{\lambda}{\gamma_f \beta}(\sigma_D + \phi)x_t - \frac{\gamma_b}{\beta \gamma_f}\pi_{H,t-1} \quad (12)$$

In sensitivity analysis with the above model simulations are stable, even with low weights on inflation. Since a positive smoothing parameter q_i reduces the policy response to inflation some weight on π is required. With $q_i = 0$ even no weight on inflation generates stable outcomes. For example, if $q_i = 1$ outcomes are indeterminate with $q_x = 0$ and q_π less than 1; they are also indeterminate with $q_x =0.07$ if q_π less than 0.9; but if $q_i =0$ and $q_x =0.07$ outcomes are stable even with $q_\pi=0$. In the estimated reaction functions of the simulations in Goyal (2011a, Table 4) the weights on inflation range from 4.28 to 0.0091[5]. The lags in the system, and other structural aspects, may be contributing to stability even with a low policy reaction to inflation.

Since the SOEME model under policy optimization was stable under more relaxed conditions on the parameters, we next derive stability conditions for the simulated equations.

3.1 The stability condition for the simulated model under a policy rule

The two simulation equations (11) and (12), written in the form required for stability analysis are (derived in Appendix A):

$$x_t = \Omega\left[\sigma_D E_t X_{t+1} - \left(\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f \beta} + 0.8\phi_\pi \gamma_b\right)\pi_{H,t-1} + 0.2\phi_x x_{t-1} + 0.2 v_{t-1} + \right.\right. \quad (13)$$
$$\left.\left.(0.8\phi_\pi \gamma_f \beta + 1)E_t \pi_{H,t+1} + 0.8\rho v_{t-1} - \overline{rr}_t\right)\right]$$

[5] A reaction function differs from the objective function or policy rule in that it gives the final weight on the CB objectives after the constraints subject to which the optimization is done, or to which the rule is applied, are taken into account.

$$\pi_{H,t} = \Omega\left(\gamma_f \beta - \lambda(\sigma_D + \phi)(0.8\phi_\pi \gamma_f \beta + 1)\right)E_t \pi_{H,t+1} + \Omega\lambda(\sigma_D + \phi)\sigma_D E_t x_{t+1} -$$
$$\left[\Omega\lambda(\sigma_D + \phi)\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f \beta} + 0.8\phi_\pi \gamma_b\right) + \gamma_b\right]\pi_{H,t-1} - \Omega\lambda(\sigma_D + \phi)0.2\phi_x x_{t-1} - \tag{14}$$
$$\Omega\lambda(\sigma_D + \phi)\left(0.2v_{t-1} + 0.8\rho v_{t-1} - \overline{rr_t}\right)$$

They give the following higher order difference equation system:

$$\begin{bmatrix} x_t \\ \pi_{H,t} \end{bmatrix} = \Omega A \begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{H,t+1} \end{bmatrix} - \Omega B \begin{bmatrix} x_{t-1} \\ \pi_{H,t-1} \end{bmatrix} + \Omega C\left[\overline{rr}_t - 0.2v_{t-1} - 0.8\rho v_{t-1}\right] \tag{15}$$

$$A = \begin{bmatrix} \sigma_D & -(0.8\phi_\pi \gamma_f \beta + 1) \\ \lambda(\sigma_D + \phi)\sigma_D & \gamma_f \beta - \lambda(\sigma_D + \phi)(0.8\phi_\pi \gamma_f \beta + 1) \end{bmatrix}$$

$$B = \begin{bmatrix} 0.2\phi_x & -\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f \beta} + 0.8\phi_\pi \gamma_b\right) \\ \lambda(\sigma_D + \phi)0.2\phi_x & \lambda(\sigma_D + \phi)\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f \beta} + 0.8\phi_\pi \gamma_b\right) + \gamma_b \end{bmatrix}$$

$$C = \begin{bmatrix} 1 \\ \lambda(\sigma_D + \phi) \end{bmatrix}$$

Given that both the output gap and inflation are non-predetermined variables, the solution to above system is locally unique if and only if the following three conditions are satisfied[6].

$$1 + a_1 + a_2 > 0 \tag{16}$$

$$1 - a_1 + a_2 > 0 \text{ or } 1 + a_1 > a_2 \tag{17}$$

$$a_2 < 1 \tag{18}$$

The first condition can be stated as 1 + AΩ - BΩ > 0 and solves to (see Appendix B):

$$(\sigma_D \gamma_f \beta - 0.2\phi_x \gamma_b)\Omega > -1 \tag{19}$$

[6] Discussed in Bullard and Mitra (2002), Blanchard and Kahn (1980), Woodford (2003), Gali (2008).

For the second condition $(A+B)\Omega<1$, and implies:

$$\left(0.2\phi_x \gamma_b + \sigma_D \gamma_f \beta\right)\Omega < 1 \tag{20}$$

The third condition, $-B\Omega<1$, is trivially true.

Conditions (19) and (20) imply there is no constraint on the weight given to inflation. Both give high upper bounds for ϕ_x with the bound from (20) being lower, so that it is the operative condition [7]. The stability conditions are all satisfied for the parameters in Table 1.

3.2 Stability introducing government debt in the simulation model

There are interesting implications for stability when government debt is introduced in the AD and AS model. Deficits and interest payments add to government debt. Following Goyal (2011b) assume a cashless economy in which all government debt consists of riskless one-period nominal debt. The maturity value of nominal government debt, B_tP_t changes over time as follows:

$$B_t P_t = \left(1 + i_t\right) B_{t-1} P_{t-1} + \left(P_t G_t - T_t\right) \tag{21}$$

The maturity value of real public debt is B_t. Real government purchases are G_t and nominal net tax collections are T_t so that real tax collections are $\tau \equiv T_t / P_t$. The real debt to output ratio is b_t. Assuming a positive rate of growth of output g and positive inflation $1+g_t = Y_t / Y_{t-1}$, $1+\pi_t = P_t / P_{t-1}$ and making other transformations gives:

$$b_t - b_{t-1} = \left(i_t - \pi_t - g_t\right) b_{t-1} + \frac{G_t}{Y_t} - \frac{\tau_t}{Y_t} \tag{22}$$

Real debt rises with the nominal interest rate, falls with inflation and the growth rate, and rises with the primary deficit ratio $\frac{G_t}{Y_t} - \frac{\tau_t}{Y_t}$.

[7] In the benchmark calibrations with $\gamma_b = 0.2$ the upper bound for ϕ_x is 49.2 and with $\gamma_b = 0.9$ it is 16.6.

To analyze responses to macroeconomic shocks, it is necessary to linearize (22) around a steady-state. In a steady state with zero inflation and real disturbances, B_t, real tax collections, $\tau \equiv T_t / P_t$, and G are equal to steady-state values $\bar{\tau}, \bar{B}, > 0$, $\bar{G} \geq 0$, and $i_t = \bar{i} \equiv \beta^{-1} - 1 > 0$. If $Y_t = \bar{Y} > 0$ grows at a steady-state growth g, $\bar{B}$ will also grow at the same rate so $\bar{b} \equiv \bar{B}/\bar{Y}$ is constant. For consistency with the evolution of nominal debt (3), steady-state fiscal values must satisfy $\bar{\tau} = \bar{G} + (1-\beta)\bar{B}$. The linearization gives:

$$\hat{b}_t = \beta^{-1}\left[\hat{b}_{t-1} - \bar{b}\pi_t - \bar{b}g \ \ + \hat{G}_t - \hat{\tau}_t\right] + \bar{b}\hat{i}_t \tag{23}$$

Where $\hat{b}_t \equiv (B_t - \bar{B})/\bar{Y}$, $\hat{\tau}_t \equiv (\tau_t - \bar{\tau})/\bar{Y}$ and $\hat{G}_t = (G_t - \bar{G}/\bar{Y})$, and $\hat{i}_t = i_t - \bar{i}$. The term in steady-state growth g in equation (23) comes from assuming a steady-state rate of growth *g* of natural output $\overline{Y_t}$ so that $\bar{Y}_t \neq \bar{Y}_{t-1}$. Such growth is to be expected for an emerging market in the process of converging to world output levels.

For determination of a local equilibrium it is sufficient to consider fiscal rules that are nearly consistent with a steady state[8]. Woodford (2003, pp.312) defines a fiscal or tax rule as *locally Ricardian* if on substituting into the local flow budget constraint (23) "it implies that $\{b_t\}$ remains forever within a bounded neighborhood of $\bar{B}$, for all paths of the endogenous variables $\{\pi_t, Y_t, i_t\}$ that remain forever within some sufficiently small neighborhoods of the steady-state values $(0, \bar{Y}, \bar{i})$, and all small enough values of the exogenous disturbances (including $\hat{G}_t$)." With this condition, the fiscal policy rule can be neglected, since the monetary policy rule and the outcomes of equilibrium inflation, output and interest rates do not depend on the paths of either of the purely fiscal variables $\{B_t, \tau_t\}$. They cancel out in the individual's budget constraint.

Consider a linear approximation (24) to a tax rule where τ_b and τ_g are the respective response coefficients of taxes to deviations in debt ratio and in government expenditure:

[8] The treatment in this section follows Woodford (2003), Chapter 4, Section 4, and Goyal (2011b).

$$\hat{\tau}_t = \tau_b \hat{b}_{t-1} + \tau_g \hat{G}_t \tag{24}$$

Substituting this into (23) gives a law of motion for real government debt (25).

$$\hat{b}_t = \beta^{-1}\left[(1-\tau_b)\hat{b}_{t-1} - \bar{b}\pi_t - \bar{b}g + (1-\tau_g)\hat{G}_t\right] + \bar{b}\hat{i}_t \tag{25}$$

The latter is stable or the tax rule (24) is locally Ricardian if and only if:

$$\left|\beta^{-1}(1-\tau_b)\right| < 1 \tag{26}$$

Or if $\tau_b \le 1$, then fiscal policy or the tax rule is locally Ricardian if and only if $\tau_b > 1-\beta$. Woodford (2003) also shows if fiscal policy is locally Ricardian, equilibrium is determinate if and only if the response of monetary policy to inflation exceeds unity, where an equilibrium is "(locally) *determinate* if and only if there are unique bounded equilibrium processes for all of the endogenous variables $\{b_t, \pi_t, Y_t, i_t\}$ for sufficiently tightly bounded processes for the exogenous disturbances (pp. 314)".

We define a new concept of semi-stable equilibrium:

Definition: Equilibrium is semi stable if and only if there are unique converging processes for all the endogenous variables for large but bounded exogenous disturbances. It differs from Woodfords definition in that the bounds on the equilibrium processes for the endogenous variables can be large.

The AD, AS system plus the evolution of debt (25) require the additional stability condition (26).

Formal stability analysis also demonstrates this. Writing the AD, AS system of equations as:

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{H,t+1} \end{bmatrix} - \frac{1}{\Omega A}\begin{bmatrix} x_t \\ \pi_{H,t} \end{bmatrix} - \frac{B}{A}\begin{bmatrix} x_{t-1} \\ \pi_{H,t-1} \end{bmatrix} = C_t \tag{27}$$

Let:

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{H,t+1} \end{bmatrix} = E_t z_{t+1}, \quad D = -\begin{bmatrix} 1 & B \\ \Omega A & A \end{bmatrix}, \quad K_t = [z_t \ z_{t-1}]$$

Adding the debt equation gives a system of equations of the form:

$$\begin{bmatrix} E_t z_{t+1} \\ \hat{b}_t \end{bmatrix} = \begin{bmatrix} D & 0 \\ c' & \beta^{-1}(1-\tau_b) \end{bmatrix} \begin{bmatrix} K_t \\ \hat{b}_{t-1} \end{bmatrix} + C_t \tag{28}$$

Where c' just includes $\bar{b}$ and vector of zeroes.

Because the matrix is block diagonal, its eigenvalues are the eigenvalues of D and the diagonal element $\beta^{-1}(1-\tau_b)$. Therefore determinacy requires eigenvalues of D in mod form should be outside the unit circle and the value of $\beta^{-1}(1-\tau_b)$ should be inside the unit circle. The latter is the case of a locally Ricardian fiscal rule[9].

If fiscal policy is locally non-Ricardian, bounded paths for the endogenous variables will require monetary policy to violate the Taylor Principle and moderate its response to inflation in order to prevent government debt from exploding. So unsustainable borrowing will require monetary accommodation.

4. Response to shocks

Since the SOEME simulation model with a monetary policy rule is stable, it is possible to analytically derive the response to monetary shocks v_t using the method of undetermined coefficients (derived in Appendix C):

$$\pi_{H,t} = \Theta v_{t-1} \text{ where } \Theta = \frac{(0.2 + 0.8\rho)}{\Omega\left(\frac{0.2\phi_x \gamma_f \beta}{\lambda(\sigma_D + \phi)} - \frac{\sigma_D \gamma_b}{\lambda(\sigma_D + \phi)} - \frac{1}{\lambda(\sigma_D + \phi)}\right)} \tag{29}$$

$$x_t = \frac{1}{\lambda(\sigma_D + \phi)}\left(\Theta v_{t-1} - \gamma_b \Theta v_{t-2} - \gamma_f \beta \Theta v_t\right) \tag{30}$$

[9] In benchmark simulations (Goyal 2010), the value of $\beta^{-1}(1-\tau_b) = 1.01 * 0.15 = 0.8585$, so the fiscal rule is Ricardian.

With the benchmark parameters of Table 1, a persistent rise in v_t (monetary tightening) of 0.1 leads to a fall in inflation of 0.03 and in output of 0.000565. This compares well with a natural rate shock in the simulations, which raises the policy rate by 0.013 and reduces inflation by 0.0126 and output by 0.0126 in the first period. The effect is stronger under optimization but in the same direction as with a policy rule.

A key result, supported by the analysis and the simulations, is: lags and rigidities in the SOEME, make it stable for a coefficient of inflation, in both the loss function and the policy rule, of less than unity.

4.1 Simulated responses in the SOEME debt model

The additional parameters in the simulated SOEME debt model are also given in Table 1. They are calibrated on Indian data[10]. The effects of shocks to domestic inflation, to the natural rate, and to government expenditure were explored in Goyal (2011b). Each shock was of the same generic form. The $\hat{G}$ shock, for example, can be written as $\hat{G}_t = \rho^G \hat{G}_{t-1} + \varepsilon_t^G$.

The cost shock has the least persistence and its effects therefore were the most transient. Inflation following the cost shock led to a fall in $\hat{b}_t$ but the system was back at its steady state value in 4 periods.

If natural rates fell, $\hat{b}_t$ rose sharply, as the government borrowed against the rise in potential output due to a positive supply shock, or spent to maintain demand after a negative demand shock, or compensated for a fall in c_P. These are all factors reducing natural rates. This fiscal response dominated the reduction in $\hat{b}$ due to the fall in interest rates, which equation (25) shows should occur. Since the policy rate fell less than the natural rate, the output gap rose, explaining some of the adjustments. Convergence back to the steady state was slow, not fully completed in the 12 periods.

[10] The calibrations are explained and detailed time paths and figures of the simulations are given in Goyal (2011b).

Sensitivity analysis for both types of shocks was similar except for the response to changes in g. A shock to the consumption of the poor induced the government to borrow more under higher growth, but the reduction in the debt ratio under higher growth reduced the deviation of debt under a cost shock. A fall in τ_b reduced the deviation from steady state debt, since adjustment back would be more difficult. A lower $\bar{b}$, however did not lead to a rise in borrowing as could be expected if steady-state debt was low. Instead, as lower servicing costs reduced current borrowing requirement, $\hat{b}_t$ rose less.

In accordance with theoretical results on response to inflation, equilibrium did not exist if q_π <1 (Figures 1 and 2). But under both cost and natural rate shocks, if q_π =1.1 instead of the benchmark 2, equilibrium existed with lower policy rates. This monetary accommodation reduced the change in debt, since debt rises with interest rates. But deviations in the other macroeconomic variables increased because of the lower policy response.

Since, in a model with debt the forward-looking component of behavior increases, q_π has to be higher than it is in the optimizing SOEME model for equilibrium to exist. But the leeway in stability that results from the lags and other structural aspects such as a positive steady-state growth built in, shows up in the SOEME debt model in more relaxed restrictions on τ_b. The relaxations occur partly since the fall in τ_b reduces the initial deviation in debt.

We define equilibrium to be semi-stable when the initial deviation is high but adjustment leads towards the steady-state, even though the steady-state may not be reached in the period of our simulations. For highly persistent shocks, equilibria are semi-stable. These equilibria are still determinate according to Woodfords definition since his qualifier of 'reasonable shock' is violated.

Table 2 summarizes some simulation results for the sensitivity analysis of monetary (natural rate) and fiscal (government expenditure) shocks. The natural rate shock simulations are labeled N. In N1 the only change from benchmark shock is $\tau_b = 0$, equilibrium is determinate, the other macroeconomic variables are the same as in the benchmark shock, only the deviation in debt is considerably reduced. Similarly in N2 where τ_b is lower than the benchmark but still positive. In simulation N3 where all the weights in the CB's loss function are reduced to 0.5 and τ_b is put at zero, equilibrium is determinate. The macroeconomic adjustment is of a similar order of magnitude as in N4 where q_π reduced to 1.1 from 2 but the initial rise in debt is much lower for N3 compared to N4. In N5, when steady-state debt $\bar{b}$ is increased, the deviation of debt rises. In N6, a fall in output growth, steeply raises the deviation of debt.

With a calibrated 0.01 shock to $\hat{G}$ of persistence 0.25 the only effect is a temporary fall in debt (G1). But equilibrium is indeterminate if persistence exceeds 0.9. With persistence of 0.85 equilibrium becomes determinate but semi stable. Equilibrium is indeterminate if q_π <1, determinate if q_π =1.1 and above. But volatility in macroeconomic variables including debt is so high as to verge on the unstable, for $\hat{G}$ of persistence 0.85 (G2 and G3). This is so even for the benchmark $\hat{G}$ shock q_π =2 (G3). But the volatility of macroeconomic variables is reduced if τ_b = 0 since the initial rise in debt is considerably reduced. Even so, $\hat{b}_t$ remains far from steady-state values at the end of 12 periods. Figures 3 and 4 show, that equilibria are semi stable for both the highly persistent N and G shocks since debt does tend towards the steady-state even if slowly.

Figures 1 and 2 show the regions of indeterminacy and stability in q_π and q_x space. Equilibria are unstable for low values of q_π under a natural rate shock. For a cost shock the weight required on q_π for stability rises as q_x rises. If the weight on q_π is as low as 1.1 or 1.2 equilibrium becomes indeterminate for 0 value of q_x.

We have kept q_i=1 to capture CB's preference for small interest rate changes. But since a large q_i reduces the interest rate response, it can make equilibrium indeterminate.

For a cost shock, whenever $q_i \geq q_\pi > 1$ equilibrium is indeterminate. When q_π is less than 1, solution exists only when $q_i \leq q_\pi < 1$.

For a natural rate shock indeterminacy exist whenever $q_i \geq q_\pi > 1$. Semi unstable equilibrium requires $q_i < q_\pi$. When $q_\pi < 1$, equilibrium exists iff $q_i \leq q_\pi < 1$

As in simulations with the SOEME AD AS model (Goyal 2011), a rise in the share of the rich and in openness reduces initial interest and therefore debt response, for both types of shocks, implying greater debt volatility is to be expected in a poorer less globally integrated country.

As for the SOEME, lags and rigidities allow relaxations in the stability conditions. The concept of semi-stability captures the slow yet persistent adjustments to equilibrium.

We have obtained analytical results on the policy response coefficients consistent with stability in emerging models. We next compare the estimated Indian monetary policy rule to these results.

5. Estimated monetary policy rule

We worked with a monetary-policy rule in deriving the stability results. Woodford's (2001) result was that interest rate rules lead to indeterminacy of the rational expectations price level only if the path of the short-term policy rate is exogenous. In particular, in his simple optimizing model, determinacy required a feedback from inflation greater than one. This is known as the Taylor Principle—for each one-percent increase in inflation, the central bank should raise the nominal interest rate by more than one percentage point (Taylor 1993). Our results suggest that in more complex models with different types of lags and rigidities the feedback coefficients required for determinacy can be very different.

There is also a large empirical literature estimating the Taylor rule. The original equation was:

$$i_t = \pi_t + r_t^* + \phi_\pi\left(\pi_t - \pi_t^*\right) + \phi_x\left(y_t - \bar{y}_t\right) \tag{31}$$

Where, π_t^* is the desired rate of inflation, r_t^* is the assumed equilibrium real interest rate, y_t, is the logarithm of real GDP, and $\bar{y}_t$ is the logarithm of potential output, as determined by a linear trend. Taylor proposed setting $a_\pi = a_x = 0.5$. As long as $a_\pi > 0$, an increase in inflation of one percentage point would lead the CB to raise the nominal interest rate by $1 + a_\pi$, thus raising the real interest rate. The simple NKE models can imply a very low a_x, since in forward-looking models with demand shocks the feedback to inflation is sufficient to stabilize output[11]. In our calibrated model the only feedback condition required for stability is that the feedback from the output gap must not be too large.

As the empirical Taylor rule literature developed, the estimated equation was simplified. Either the short policy rate was regressed on the deviation of output from potential and of inflation from target, or a constant term was assumed to include a constant inflation target and real interest rate. So the short policy rate was regressed on inflation and on the deviation of output from potential including a constant capturing the inflation target. A lagged interest rate was included to capture policy smoothing. We estimate the latter Taylor rule specification for India to compare its coefficients with the optimizing results, and assess Indian monetary policy.

For the short-term policy rate, we use data at quarterly frequency from 2000Q2 to 2011Q2. Variables include call or money market rate, GDP and wholesale price index in two forms core inflation and headline inflation. All the variables are tested for seasonality. Analysis of linear plots suggests that quarterly GDP and WPI series have multiplicative seasonality. Hence we de-seasonalize the series using the X-12 ARIMA procedure. Output gap was derived using the HP filter for measuring trend output i.e. the actual output gap is calculated as the percent deviation of real GDP from a target, as it was originally proposed by Taylor:

[11] The NKE literature calls it the 'divine coincidence' when the CB does not need to take fluctuations in the output gap into account when setting interest rates. Woodford's (2001) differences with the empirical Taylor rule were: First, the welfare theoretic loss function implies the inflation target should be zero in the pure frictionless model. Second the output gap should be calculated using the natural output, not the past deterministic trend. All the shocks, such as technology, consumption of the P-type and world income, that affect the natural interest rate in equation (2) affect the natural output. See Goyal (2009) for more details on natural output in a SOEME.

$$y = ((Y-Y^*)/Y^*)*100$$

where Y is real GDP (or more often industrial production index), and Y*is trend real GDP given by HP filter. Year-on-year inflation is measured using annual percentage change in Wholesale Price Index (WPI) and defined as headline inflation and core inflation is defined as nonfood, manufacturing goods inflation, whose share is around 52.2 percent in WPI. All the variables (growth rate and inflation terms) are in percentages, following the practice in the literature[12].

Unit root tests, i.e. Augmented Dickey-Fuller test results suggest that all the variables are stationary. Durbin Watson test suggest presence of serial correlation and Breusch-Pagan test shows the presence of heteroskedasticity in the error terms. Hence, we estimate our model using ordinary least squares regression with Newey-West variance-covariance matrix, in order to correct for both autocorrelation and heteroskedasticity.

The two estimated equations with headline inflation and core inflation (t-values in brackets) are as follows:

(1) Headline inflation

$$\begin{aligned} r_t &= 1.85 \;+\; 0.58 r_{t-1} \;+\; 0.156 \pi_t \;+\; 0.32 y_t \\ &\quad (2.71) \quad (5.24) \qquad (2.83) \qquad (3.12) \end{aligned}$$

(2) Core inflation

$$\begin{aligned} r_t &= 2.12 \;+\; 0.59 r_{t-1} \;+\; 0.126 \pi_t \;+\; 0.29 y_t \\ &\quad (2.96) \quad (5.21) \qquad (2.06) \qquad (2.93) \end{aligned}$$

The coefficients are a similar order of magnitude to the reaction functions estimated in the simulations. The results suggest Indian policy makers have implicitly understood optimality and context in that the response to both inflation and the output gap is not high. The exchange rate was market determined in this period. Calibrations

[12] See Hamilton et.al. (2011), Aleksandra Maslowska (2010), Michael Hutchison et.al. (2009).

in Goyal (2011a) suggest the feedback coefficients could be even lower, if the real exchange rate were stable. The coefficients are also consistent with stability.

6. Conclusion

Most estimations of Taylor rules for emerging markets, including ours in this paper, give a coefficient for inflation of much below unity. The NKE literature shows a coefficient of above unity can impose stability in optimizing models with forward-looking behaviour. Nominal rigidities in NKE models alone do not provide an adequate nominal anchor to prevent explosive outcomes. These theoretical stability results turn out to be the same in an NKE DSGE model adapted to an open economy emerging market model with two types of agents to capture heterogeneity in labour markets and consumers.

But in the simulated version of the optimizing model that allows lagged policy rates to enter the aggregate demand equation, and has some degree of backward looking behaviour, the derived stability condition does not impose any restriction on the coefficient of inflation in a policy rule, rather it imposes a cap on the coefficient of output. The reaction functions estimated in optimizing simulations are consistent with this rule, as are estimated coefficients of Taylor rules for India.

As further consistency checks, analytical solutions to monetary policy shocks using the simulation equations, give results similar to the simulations. Therefore a key result, supported by analysis, simulations and estimations is: lags and rigidities in the SOEME, make it stable for a coefficient of inflation, in Central Bank loss functions, simulation reaction functions as well as the policy rule, of less than unity.

Outcomes are stable even with a weight of zero on inflation in the loss function when there is no weight on interest rate smoothing, and weights on inflation in estimated reaction functions can be very low. The lags in the system, and other structural aspects, may be contributing to stability even with a low policy reaction to inflation.

Since, in the SOEME with debt the forward-looking component of behavior increases, the weight on inflation in the loss function has to be higher than it is in the optimizing SOEME model for equilibrium to exist. But the leeway in stability that results from the lags and other structural aspects such as a positive steady-state growth, shows up in the SOEME debt model in more relaxed restrictions on taxes in response to expenditure for stability. The relaxations occur partly since a fall in tax rates reduces the initial rise in debt.

The results suggest, more generally, that the effect of specific rigidities on stability should be more carefully explored, and knowledge of the specific rigidities in an economy can give useful inputs for the design of policy.

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Appendix A

To derive equations (13) and (14) for the simulation model for a monetary policy rule:

$$x_{t+1} = \left[1 + \frac{\lambda}{\gamma_f \beta}\left(1 + \frac{\phi}{\sigma_D}\right)\right] x_t + \frac{1}{\sigma_D}\left(0.2r_{t-1} + 0.8r_t + \frac{\pi_{H,t}}{\gamma_f \beta} + \frac{\gamma_b}{\gamma_f \beta}\pi_{H,t-1} - \overline{rr_t}\right) \quad \text{(A1)}$$

$$\pi_{H,t+1} = \frac{1}{\gamma_f \beta}\pi_{H,t} - \frac{\lambda}{\gamma_f \beta}(\sigma_D + \phi)x_t - \frac{\gamma_b}{\gamma_f \beta}\pi_{H,t-1} \quad \text{(A2)}$$

$$r_t = \rho + \phi_\pi \pi_{H,t} + \phi_x x_t + v_t \quad \text{(A3)}$$

$$v_t = \rho_v v_{t-1} + \varepsilon_t \quad \text{(A4)}$$

$$\left[1+\frac{\lambda}{\gamma_f\beta}\left(1+\frac{\phi}{\sigma_D}\right)\right]x_t = x_{t+1} - \frac{1}{\sigma_D}\left(0.2r_{t-1} + 0.8r_t + \frac{\pi_{H,t}}{\gamma_f\beta} + \frac{\gamma_b}{\gamma_f\beta}\pi_{H,t-1} - \overline{rr_t}\right) \qquad \text{(A5)}$$

Or,

$$\left[1+\frac{\lambda}{\gamma_f\beta}\left(1+\frac{\phi}{\sigma_D}\right)\right]\sigma_D x_t = \sigma_D x_{t+1} - \left(0.2r_{t-1} + 0.8r_t + \frac{\pi_{H,t}}{\gamma_f\beta} + \frac{\gamma_b}{\gamma_f\beta}\pi_{H,t-1} - \overline{rr_t}\right)$$

Substituting for r_{t-1} and r_t from equation (A3) we get:

$$\left[1+\frac{\lambda}{\gamma_f\beta}\left(1+\frac{\phi}{\sigma_D}\right)\right]\sigma_D x_t = \sigma_D x_{t+1} - \Big(0.2\rho + 0.2\phi_\pi\pi_{H,t-1} + 0.2\phi_x x_{t-1} + 0.2v_{t-1} +$$

$$0.8\rho + 0.8\phi_\pi\pi_{H,t-1} + 0.8\phi_x x_{t-1} + 0.8v_t \frac{\pi_{H,t}}{\gamma_f\beta} + \frac{\gamma_b}{\gamma_f\beta}\pi_{H,t-1} - \overline{rr_t}\Big)$$

$$\left[1+\frac{\lambda}{\gamma_f\beta}\left(1+\frac{\phi}{\sigma_D}\right)\right]\sigma_D x_t = \sigma_D x_{t+1} - \left(\rho + \left(0.2\phi_\pi + \frac{\gamma_b}{\gamma_f\beta}\right)\pi_{H,t-1} + 0.2\phi_x x_{t-1} + 0.2v_{t-1} +\right.$$

$$\left.(0.8\phi_\pi\gamma_f\beta + 1)\frac{\pi_{H,t}}{\gamma_f\beta} + 0.8\phi_x x_t + 0.8v_t - \overline{rr_t}\right)$$

Substituting for $\frac{\pi_{H,t}}{\gamma_f\beta}$ from equation A2, that is:

$$\frac{1}{\gamma_f\beta}\pi_{H,t} = \pi_{H,t+1} + \frac{\lambda}{\gamma_f\beta}(\sigma_D + \phi)x_t + \frac{\gamma_b}{\gamma_f\beta}\pi_{H,t-1}$$

Solving for x_t:

$$\left\{\left[1+\frac{\lambda}{\gamma_f\beta}\left(1+\frac{\phi}{\sigma_D}\right)\right]\sigma_D + 0.8\phi_x\right\}x_t = \sigma_D x_{t-1} - \left(\left(0.2\phi_\pi + \frac{\gamma_b}{\gamma_f\beta}\right)\pi_{H,t-1} + 0.2\phi_x x_{t-1} + 0.2v_{t-1} +\right.$$

$$\left.(0.8\phi_\pi\gamma_f\beta + 1)\left\{\pi_{H,t+1} + \frac{\lambda}{\gamma_f\beta}(\sigma_D + \phi)x_t + \frac{\gamma_b}{\gamma_f\beta}\pi_{H,t-1}\right\} + 0.8v_t - \overline{rr}_t\right)$$

or,

$$\left\{\left[1+\frac{\lambda}{\gamma_f\beta}\left(1+\frac{\phi}{\sigma_D}\right)\right]\sigma_D+0.8\phi_x+\left(0.8\phi_\pi\gamma_f\beta+1\right)\frac{\lambda}{\gamma_f\beta}(\sigma_D+\phi)\right\}x_t=\sigma_D x_{t-1}-$$
$$\left(\left(0.2\phi_\pi+\frac{\gamma_b}{\gamma_f\beta}\right)\pi_{H,t-1}+0.2\phi_x x_{t-1}+0.2v_{t-1}+\left(0.8\phi_\pi\gamma_f\beta+1\right)\left\{\pi_{H,t+1}+\frac{\gamma_b}{\gamma_f\beta}\pi_{H,t-1}\right\}+0.8v_t-\overline{rr}_t\right)$$

or,

$$\left\{\left[1+\frac{\lambda}{\gamma_f\beta}\left(1+\frac{\phi}{\sigma_D}\right)\right]\sigma_D+0.8\phi_x+\left(0.8\phi_\pi\gamma_f\beta+1\right)\frac{\lambda}{\gamma_f\beta}(\sigma_D+\phi)\right\}x_t=\sigma_D x_{t-1}-$$
$$\left(\left(0.2\phi_\pi+\frac{2\gamma_b}{\gamma_f\beta}+0.8\phi_\pi\gamma_b\right)\pi_{H,t-1}+0.2\phi_x x_{t-1}+0.2v_{t-1}+\left(0.8\phi_\pi\gamma_f\beta+1\right)\pi_{H,t+1}+0.8v_t-\overline{rr}_t\right) \quad \text{(A6)}$$

Taking expectations on both sides and setting:

$$\Omega=\frac{1}{\left[1+\frac{\lambda}{\gamma_f\beta}\left(1+\frac{\phi}{\sigma_D}\right)\right]\sigma_D+0.8\phi_x+\left(0.8\phi_\pi\gamma_f\beta+1\right)\frac{\lambda}{\gamma_f\beta}(\sigma_D+\phi)}$$

Gives equation (13) in the text.

To solve for the AS equation, substitute equation (A6) in the equation given below:

$$\pi_{H,t}=\gamma_f\beta\pi_{H,t+1}+\lambda(\sigma_D+\phi)x_t+\lambda_b\pi_{H,t-1}$$

$$\pi_{H,t}=\gamma_f\beta\pi_{H,t+1}+\lambda(\sigma_D+\phi)\Omega\Big[\sigma_D x_{t+1}-\Big(\left(0.2\phi_\pi+\frac{2\gamma_b}{\gamma_f\beta}+0.8\phi_\pi\gamma_b\right)\pi_{H,t-1}+0.2\phi_x x_{t-1}+$$
$$0.2v_{t-1}+\left(0.8\phi_\pi\gamma_f\beta+1\right)\pi_{H,t+1}+0.8v_t-\overline{rr}_t\Big)\Big]+\gamma_b\pi_{H,t-1}$$

$$\pi_{H,t}=\Omega\left(\gamma_f\beta-\lambda(\sigma_D+\phi)\left(0.8\phi_\pi\gamma_f\beta+1\right)\right)\pi_{H,t+1}+\Omega\lambda(\sigma_D+\phi)\sigma_D x_{t+1}-$$
$$\left[\Omega\lambda(\sigma_D+\phi)\left(0.2\phi_\pi+\frac{2\gamma_b}{\gamma_f\beta}+0.8\phi_\pi\gamma_b\right)+\gamma_b\right]\pi_{H,t-1}-\Omega\lambda(\sigma_D+\phi)0.2\phi_x x_{t-1}-$$
$$\Omega\lambda(\sigma_D+\phi)\left(0.2v_{t-1}+0.8v_t-\overline{rr}_t\right)$$

Taking expectation on both sides gives equation (14).

Appendix B

To derive the stability conditions for the simulation model equations with a policy rule:

The first stability condition can be stated as:

$$1 + A\Omega - B\Omega > 0$$

$$(A - B)\Omega > -1$$

$$\frac{1}{\Omega} < \sigma_D\left[\gamma_f\beta - \lambda(\sigma_D + \phi)(0.8\phi_\pi\gamma_f\beta + 1)\right] + \lambda\sigma_D(\sigma_D + \phi)(0.8\phi_\pi\gamma_f\beta + 1) - 0.2\phi_x$$
$$\left[\lambda(\sigma_D + \phi)\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f\beta} + 0.8\phi_\pi\gamma_b\right) + \gamma_b\right] + \lambda 0.2\phi_x(\sigma_D + \phi)\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f\beta} + 0.8\phi_\pi\gamma_b\right)$$

$$\frac{1}{\Omega} < \sigma_D\left[\gamma_f\beta - \lambda(\sigma_D + \phi)(0.8\phi_\pi\gamma_f\beta + 1)\right] + \lambda\sigma_D(\sigma_D + \phi)(0.8\phi_\pi\gamma_f\beta + 1) - 0.2\phi_x$$
$$\lambda(\sigma_D + \phi)\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f\beta} + 0.8\phi_\pi\gamma_b\right) - 0.2\phi_x\gamma_b + \lambda 0.2\phi_x(\sigma_D + \phi)\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f\beta} + 0.8\phi_\pi\gamma_b\right)$$

or,

$$\frac{1}{\Omega} < \sigma_D\gamma_f\beta - 0.2\phi_x\gamma_b$$

The first condition implies, $(\sigma_D\gamma_f\beta - 0.2\phi_x\gamma_b)\Omega > -1$

The second condition is:

$$1 - A\Omega - B\Omega > 0$$

$$(A + B)\Omega < 1$$

$$\frac{1}{\Omega} > 0.2\phi_x\left[\lambda(\sigma_D + \phi)\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f\beta} + 0.8\phi_\pi\gamma_b\right) + \gamma_b\right] - \lambda 0.2\phi_x(\sigma_D + \phi)\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f\beta} + 0.8\phi_\pi\gamma_b\right) -$$
$$\sigma_D\left[\gamma_f\beta - \lambda(\sigma_D + \phi)(0.8\phi_\pi\gamma_f\beta + 1)\right] - \lambda\sigma_D(\sigma_D + \phi)(0.8\phi_\pi\gamma_f\beta + 1)$$

or,

$$\frac{1}{\Omega} > \sigma_D 0.2\phi_x \lambda(\sigma_D + \phi)\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f \beta} + 0.8\phi_\pi \gamma_b\right) + 0.2\phi_x \gamma_b - \lambda 0.2\phi_x(\sigma_D + \phi)\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f \beta} + 0.8\phi_\pi \gamma_b\right) + \sigma_D \gamma_f \beta + \lambda\sigma_D(\sigma_D + \phi)(0.8\phi_\pi \gamma_f \beta + 1) - \lambda\sigma_D(\sigma_D + \phi)(0.8\phi_\pi \gamma_f \beta + 1)$$

or,

$$\frac{1}{\Omega} > 0.2\phi_x \gamma_b + \sigma_D \gamma_f \beta$$

or,

$$1 > (0.2\phi_x \gamma_b + \sigma_D \gamma_f \beta)\Omega$$

Therefore second condition requires $(0.2\phi_x \gamma_b + \sigma_D \gamma_f \beta)\Omega < 1$

Appendix C

To derive the response to monetary shocks in the simulation model with a monetary policy rule:

Assuming $\overline{rr}_t = 0$ and guessing that the solution will take form:

$$\begin{aligned} x_t &= \Psi_{1x} v_t + \Psi_{2x} v_{t-1} \\ \pi_{H,t} &= \Psi_{1\pi} v_t + \Psi_{2\pi} v_{t-1} \end{aligned} \tag{C1}$$

Imposing the guessed solution on equation (13) and (14) and using the method of undetermined coefficients, we can solve for the Ψs. Write the AS equation (3) as:

$$\pi_{H,t+1} = \frac{1}{\gamma_f \beta}\pi_{H,t} - \frac{\lambda}{\gamma_f \beta}(\sigma_D + \phi)x_t - \frac{\gamma_b}{\gamma_f \beta}\pi_{H,t-1} \tag{C2}$$

Using C2 we have:

$$x_t = \frac{1}{\lambda(\sigma_D + \phi)}\left(\pi_{H,t} - \gamma_b \pi_{H,t-1} - \gamma_f \beta E_t \pi_{H,t+1}\right)$$

and,

$$x_{t+1} = \frac{1}{\lambda(\sigma_D + \phi)}\left(\pi_{H,t+1} - \gamma_b \pi_{H,t} - \gamma_f \beta \pi_{H,t+2}\right)$$

Therefore,

$$E_t x_{t+1} = \frac{1}{\lambda(\sigma_D + \phi)}\left(E_t \pi_{H,t+1} - \gamma_b \pi_{H,t} - \gamma_f \beta E_t \pi_{H,t+2}\right)$$

and,

$$x_{t-1} = \frac{1}{\lambda(\sigma_D + \phi)}\left(\pi_{H,t-1} - \gamma_b \pi_{H,t-2} - \gamma_f \beta \pi_{H,t}\right)$$

By substituting these values of x_t , $x_{t-1,}$ and x_{t+1} in equation 13, we get:

$$\frac{1}{\lambda(\sigma_D + \phi)}\left(\pi_{H,t} - \gamma_b \pi_{H,t-1} - \gamma_f \beta E_t \pi_{H,t+1}\right) = \Omega[\frac{\sigma_D}{\lambda(\sigma_D + \phi)}\left(E_t \pi_{H,t+1} - \gamma_b \pi_{H,t} - \gamma_f \beta E_t \pi_{H,t+2}\right) - \left(\left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f \beta} + 0.8\phi_\pi \gamma_b\right)\pi_{H,t-1} + \frac{0.2\phi_x}{\lambda(\sigma_D + \phi)}\left(\pi_{H,t+-1} - \gamma_b \pi_{H,t-2} - \gamma_f \beta \pi_{H,t}\right) + 0.2 v_{t-1} + \left(0.8\phi_\pi \gamma_f \beta + 1\right) E_t \pi_{H,t+1} + 0.8\rho v_{t-1} - \overline{rr}_t\right)]$$

$$\frac{1}{\lambda(\sigma_D + \phi)}\left(\pi_{H,t} - \gamma_b \pi_{H,t-1} - \gamma_f \beta E_t \pi_{H,t+1}\right) = \Omega\left[\left(\frac{\sigma_D}{\lambda(\sigma_D + \phi)}\left(0.8\phi_\pi \gamma_f \beta + 1\right)\right)E_t \pi_{H,t+1} - \frac{\gamma_f \beta \sigma_D}{\lambda(\sigma_D + \phi)} E_t \pi_{H,t+2} + \left(\frac{0.2\phi_x \gamma_f \beta}{\lambda(\sigma_D + \phi)} - \frac{\sigma_D \gamma_b}{\lambda(\sigma_D + \phi)}\right)\pi_{H,t} - \left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f \beta} + 0.8\phi_\pi \gamma_b + \frac{0.2\phi_x}{\lambda(\sigma_D + \phi)}\right)\pi_{H,t-1} + \frac{0.2\phi_x \gamma_b}{\lambda(\sigma_D + \phi)}\pi_{H,t-2} - (0.2 + 0.8\rho)v_{t-1}\right]$$

$$0 = \Omega\left[\left(\frac{\sigma_D}{\lambda(\sigma_D + \phi)} + \frac{\gamma_f \beta}{\lambda(\sigma_D + \phi)} - \left(0.8\phi_\pi \gamma_f \beta + 1\right)\right)E_t \pi_{H,t+1} - \frac{\gamma_f \beta \sigma_D}{\lambda(\sigma_D + \phi)} E_t \pi_{H,t+2} + \left(\frac{0.2\phi_x \gamma_f \beta}{\lambda(\sigma_D + \phi)} - \frac{\sigma_D \gamma_b}{\lambda(\sigma_D + \phi)} - \frac{1}{\lambda(\sigma_D + \phi)}\right)\pi_{H,t} - \left(0.2\phi_\pi + \frac{2\gamma_b}{\gamma_f \beta} + 0.8\phi_\pi \gamma_b + \frac{0.2\phi_x}{\lambda(\sigma_D + \phi)} - \frac{\gamma_b}{\lambda(\sigma_D + \phi)}\right)\pi_{H,t-1} + \frac{0.2\phi_x \gamma_b}{\lambda(\sigma_D + \phi)}\pi_{H,t-2} - (0.2 + 0.8\rho)v_{t-1}\right]$$

From C1:

$$\pi_{H,t} = \Psi_{1\pi} v_t + \Psi_{2\pi} v_{t-1}$$

$$\pi_{H,t+1} = \Psi_{1\pi} v_{t+1} + \Psi_{2\pi} v_t$$

$$E_t \pi_{H,t+1} = \Psi_{1\pi} \rho v_t + \Psi_{2\pi} v_t$$

$$E_t \pi_{H,t+2} = \Psi_{1\pi} \rho^2 v_t + \Psi_{2\pi} \rho v_t$$

$$\pi_{H,t-1} = \Psi_{1\pi} v_{t-1} + \Psi_{2\pi} v_{t-2}$$

$$\pi_{H,t-2} = \Psi_{1\pi} v_{t-2} + \Psi_{2\pi} v_{t-3}$$

Substituting these values:

$$(0.2+0.8\rho)v_{t-1}=\Omega\left[\left(\frac{\sigma_D}{\lambda(\sigma_D+\phi)}+\frac{\gamma_f\beta}{\lambda(\sigma_D+\phi)}-(0.8\phi_\pi\gamma_f\beta+1)\right)(\Psi_{1\pi}\rho v_t+\Psi_{2\pi}v_t)-\right.$$

$$\frac{\gamma_f\beta\sigma_D}{\lambda(\sigma_D+\phi)}(\Psi_{1\pi}\rho^2 v_t+\Psi_{2\pi}\rho v_t)+\left(\frac{0.2\phi_x\gamma_f\beta}{\lambda(\sigma_D+\phi)}-\frac{\sigma_D\gamma_b}{\lambda(\sigma_D+\phi)}-\frac{1}{\lambda(\sigma_D+\phi)}\right)(\Psi_{1\pi}v_t+\Psi_{2\pi}v_{t-1})-$$

$$\left.\left(0.2\phi_\pi+\frac{2\gamma_b}{\gamma_f\beta}+0.8\phi_\pi\gamma_b+\frac{0.2\phi_x}{\lambda(\sigma_D+\phi)}-\frac{\gamma_b}{\lambda(\sigma_D+\phi)}\right)(\Psi_{1\pi}v_{t-1}+\Psi_{2\pi}v_{t-2})+\frac{0.2\phi_x\gamma_b}{\lambda(\sigma_D+\phi)}(\Psi_{1\pi}v_{t-2}+\Psi_{2\pi}v_{t-3})\right]$$

Since there is no term for v_t v_{t-2} or v_{t-3} their coefficients will become 0. Therefore solving for v_{t-1}, gives:

$$(0.2+0.8\rho)v_{t-1}=\Omega\left(\frac{0.2\phi_x\gamma_f\beta}{\lambda(\sigma_D+\phi)}-\frac{\sigma_D\gamma_b}{\lambda(\sigma_D+\phi)}-\frac{1}{\lambda(\sigma_D+\phi)}\right)\Psi_{2\pi}v_{t-1}-$$

$$\Omega\left(0.2\phi_\pi+\frac{2\gamma_b}{\gamma_f\beta}+0.8\phi_\pi\gamma_b+\frac{0.2\phi_x}{\lambda(\sigma_D+\phi)}-\frac{\gamma_b}{\lambda(\sigma_D+\phi)}\right)\Psi_{1\pi}v_{t-1}$$

Since, $\Psi_{1\pi}=0$ we have:

$$\Psi_{2\pi}=\frac{(0.2+0.8\rho)}{\Omega\left(\frac{0.2\phi_x\gamma_f\beta}{\lambda(\sigma_D+\phi)}-\frac{\sigma_D\gamma_b}{\lambda(\sigma_D+\phi)}-\frac{1}{\lambda(\sigma_D+\phi)}\right)}$$

or,

$$\pi_{H,t}=\frac{(0.2+0.8\rho)}{\Omega\left(\frac{0.2\phi_x\gamma_f\beta}{\lambda(\sigma_D+\phi)}-\frac{\sigma_D\gamma_b}{\lambda(\sigma_D+\phi)}-\frac{1}{\lambda(\sigma_D+\phi)}\right)}v_{t-1}$$

$$\pi_{H,t}=\Theta v_{t-1}$$

where,

$$\Theta=\frac{(0.2+0.8\rho)}{\Omega\left(\frac{0.2\phi_x\gamma_f\beta}{\lambda(\sigma_D+\phi)}-\frac{\sigma_D\gamma_b}{\lambda(\sigma_D+\phi)}-\frac{1}{\lambda(\sigma_D+\phi)}\right)}$$

From the manipulations of C1 we have:

$$x_t = \frac{1}{\lambda(\sigma_D + \phi)}\left(\pi_{H,t} - \gamma_b \pi_{H,t-1} - \gamma_f \beta \pi_{H,t+1}\right)$$

Using the above two equations we can calculate the equation for x_t

$$x_t = \frac{1}{\lambda(\sigma_D + \phi)}\left(\pi_{H,t} - \gamma_b \pi_{H,t-1} - \gamma_f \beta \pi_{H,t+1}\right)$$

or,

$$x_t = \frac{1}{\lambda(\sigma_D + \phi)}\left(\Theta v_{t-1} - \gamma_b \Theta v_{t-2} - \gamma_f \beta \Theta v_t\right)$$

Table 1: Benchmark calibrations		
Baseline Calibrations		
Degree of price stickiness	θ	0.75
Price response to output	φ	0.25
Labour supply elasticity of P type	φ_P	0.01
Labour supply elasticity of R type	φ_R	0.6
Elasticity of substitution between differentiated goods	ε	6
Steady state real interest rate or natural interest rate	ρ or $\bar{i}$	0.01
Variations in the natural interest rate due to temporary shocks	$\overline{rr}$	0.01 $\pm$
Degree of openness	α	0.3
Proportion of the R type	η	0.4
The intertemporal elasticity of substitution of the R type	$1/\sigma_R$	1
The intertemporal elasticity of substitution of the P type	$1/\sigma_P$	0
Consumption of the P type	C_p	0.2
Consumption of the R type	C_R	1
Share of backward looking inflation	γ_b	0.2
Share of forward looking inflation	γ_f	0.8
Response coefficient of taxes to the debt ratio	τ_b	0.15
Response coefficient of taxes to G expenditure	τ_g	0
Steady state public debt to output ratio	$\bar{b}$	0.8
Monthly growth rate	g	0.006
Weight of output in the CB's loss function	q_y	0.7
Weight of inflation in the CB's loss function	q_π	2
Weight of the interest rate in the CB's loss function	q_i	1
Implied parameters		
Weighted average elasticity of substitution	$1/\sigma_D$	0.58
Discount factor	β	0.99
Weighted average consumption level	C	0.75
Log deviation from world output	κ	0.1
Philips curve parameter	λ	0.24
Steady state real interest rate, discount rate	ρ	0.01
Labour supply elasticity	$1/\varphi$	4
Shocks		
Persistence of shock to G expenditure	ρ^G	0.25
Persistence of natural rate shock	ρ^r	0.75
Persistence of cost-push shock	ρ^c	0
Standard deviation of shock to G expenditure	σ_ε^G	0.1
Standard deviation of natural rate shock	σ_ε^r	0.01
Standard deviation of cost-push shock	σ_ε^c	0.2

Table 2: Simulations and volatilities

Simulations	Parameters	Standard deviations of (in percentages):				
	$\bar{b} = 0.8, \tau_b = 0.15, g = 0.006$ q_π=2, q_y=0.07, q_i =1	Consumer inflation	Output	Domestic inflation	Government debt (initial response)	Interest rate (initial response)
Cost shock	Benchmark	0.58	0.36	1.08	0.48 (-0.0174)	0.70 (0.0256)
	τ_b=0.1	0.58	0.36	1.08	0.45 (-0.0162)	0.70 (0.0256)
	g=0.008	0.58	0.36	1.08	0.47 (-0.0169)	0.70 (0.0256)
	$\bar{b}$ =0.7	0.58	0.36	1.08	0.42 (-0.0151)	0.70 (0.0256)
	q_π=1.1	0.71	0.18	1.21	0.16 (0.0056)	0.50 (0.0181)
Natural rate shock N	Benchmark	0.47	0.16	0.31	6.18 (0.2088)	0.39 (-0.0133)
N1	τ_b =0	0.47	0.16	0.31	2.20 (0.0745)	0.39 (-0.0133)
N2	τ_b =0.1	0.47	0.16	0.31	3.90 (0.1319)	0.39 (-0.0133)
N3	qs=0.5, τ_b=0	1.01	0.42	0.91	1.81 (0.0614)	0.82 (-0.0276)
N4	q_π= 1.1	1.04	0.46	0.97	5.12 (0.1730)	0.83 (-0.0279)
N5	$\bar{b}$ =0.9	0.47	0.16	0.31	7.27 (0.2456)	0.39 (-0.0133)
N6	g= 0.004	0.47	0.16	0.31	5.36 (0.1813)	0.39 (-0.0133)
$\hat{G}$ shock G1	Persistence= 0.25	0.00	0.01	0.00	1.14 (-0.0412)	0.00 (0.0000)
G2	Persistence =0.85, q_π = 1.1	80.07	14.40	42.38	251.63 (-9.6890)	37.48 (-1.4167)
G3	Persistence = 0.85	87.47	08.67	28.19	935.66 (-3.5970)	37.65 (-0.1389)
G4	Persistence = 0.85, τ_b = 0	01.15	0.12	0.42	14.43 (-0.5551)	0.56 (-0.0204)

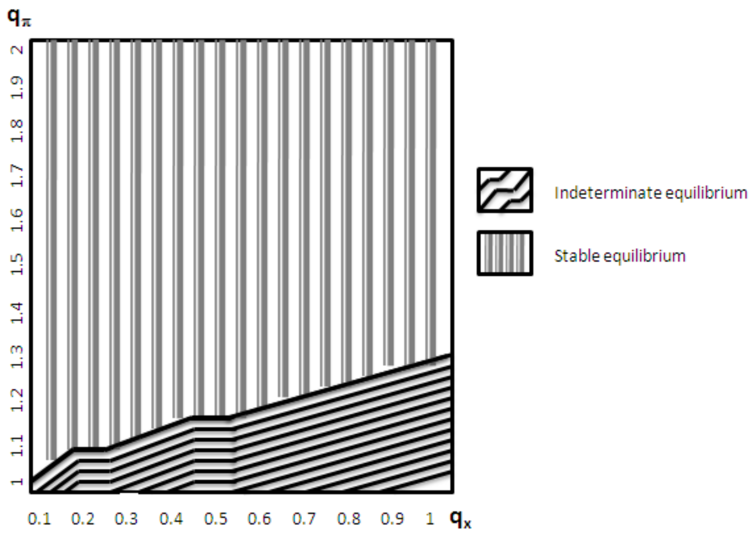

Figure 1: Cost shock

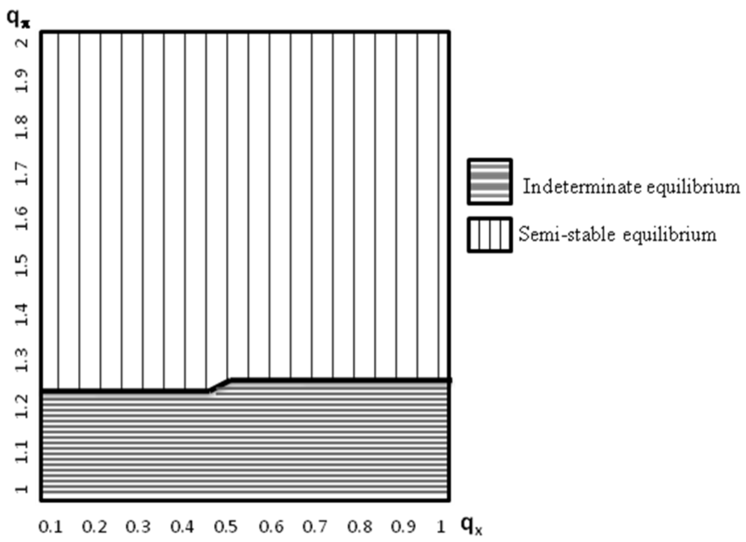

Figure 2: Natural rate shock