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Liquidity considerations in estimating implied volatility

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Abstract

Option markets have significant variation in liquidity across different option series. Illiquidity reduces the informativeness of the price. Price information for illiquid options is more noisy, and thus the implied volatilities based on these prices are more noisy. In this paper, we propose a scheme to estimate implied volatility which reduces the importance attached to illiquid options. We find that this liquidity weighted scheme outperforms conventional schemes such as the traditional vxo, or vega weights, and volatility elasticity weights.

Keywords:

liquidity, implied volatility, volatility index, Indian index options market

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Abstract

Option markets have significant variation in liquidity across different option series. Illiquidity reduces the informativeness of the price. Price information for illiquid options is more noisy, and thus the implied volatilities based on these prices are more noisy. In this paper, we propose a scheme to estimate implied volatility which reduces the importance attached to illiquid options. We find that this liquidity weighted scheme outperforms conventional schemes such as the traditional VXO, or vega weights, and volatility elasticity weights.

*Email `susant@igidr.ac.in`, URL `http://www.igidr.ac.in/faculty/susant` We are grateful to NSE for the data used in this paper. We thank Nidhi Aggarwal, Ajay Shah, Rajat Tayal and the participants of the IGIDR Finance Research Series for useful discussions. The views expressed in this paper belong to the author and not her employer.

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1 Introduction

When options markets became established and liquid, market price of options were used to directly calculate the market forecast of volatility called the "implied volatility" (IV). IV is a direct measure of the *forecast* of volatility made by economic agents. An extensive literature has documented the high quality of volatility forecasting that is embedded in IV.

Different options on the same underlying yield different values for the IV. Analytical methods are thus required to reduce multiple values for IV from different traded options on the same underlying into an efficient point estimate of IV for that underlying.

One endeavour to isolate a volatility forecast from multiple values of IV was based on developing models of option pricing that incorporated the factors that caused IV to change deterministically, such as moneyness of the option, volatility dynamics or the liquidity of the underlying.

The other strategy adopted was to develop an index of implied volatility. This was discussed extensively in the literature, which led up to the introduction of the CBOE VIX in 1993. This implied volatility index was calculated for the S&P 100 index options using the methodology proposed by Whaley (1993) and was disseminated by the CBOE in real time. The index was calculated using only at-the-money (ATM) options with a defined weighting scheme over the IV values calculated. In 2003, the CBOE shifted the VIX calculation methodology to one that used option prices over a wide range of strike values. In the following years, similar computation of an implied volatility index has commenced on numerous options exchanges worldwide. Trading in derivatives on VIX has also commenced.

The CBOE VIX methodology is predicated on all option prices being measured sharply. However, in the real world, there is substantial cross-sectional variation in the liquidity of option series. As an example, in tranquil times (September 2007), the bid-offer spread of options on the S&P 500 index at the CBOE ranged from near 0 to 200%. In turbulent times (September 2008), many more options were afflicted with illiquidity.

At present, a variety of heuristics are being utilised by exchanges worldwide in addressing this problem. In this paper, we try to frontally address the problem of illiquid options markets by constructing a weighting scheme for the construction of a volatility index, that directly incorporates the liquidity of the option. The empirical work of this paper is based on one of the most active option markets in the world: options on the NSE-50 (Nifty) index, traded at the National Stock Exchange. We use the bid-offer spread in a weighting scheme that adjusts for illiquidity when calculating the VIX. We call this the *spread adjusted* VIX, (SVIX).

We compare the performance of SVIX against three alternative weighting schemes: the 1993 CBOE index (called VXO), the vega weighted index (VVIX) and a volatility elasticity weighted index (EVIX). The performance is measured as the forecasting success of each VIX candidate against the realised volatility (RV) of the market index. The testing procedure employed is the Model Confidence Set (MCS) test (Hansen *et al.*, 2003). We find that the new SVIX that we propose is a better predictor of future RV. We also run univariate regressions of RV on each volatility index and find that while all volatility indexes contain information about future volatility, they are biased forecasts. However, the SVIX shows the smallest bias among the candidates in our test.

Option implied volatility is an important component of the information set of the financial system. The world over, options markets are being used to create implied volatility indexes using ideas similar to that of the CBOE VIX. Since all options markets have substantial cross-sectional variation in option liquidity, the ideas of this paper may potentially yield improved measurement of volatility indexes.

We present the paper as follows: Section 2 presents the issues surrounding the creation of a volatility index. It also presents the evaluation framework used to compare the performance of alternative volatility indexes. Section 3 reviews alternative schemes to construct volatility indexes. Section 4 describes the data used for the analysis. Section 5 discusses the liquidity adjusted weighting scheme, after which we present our analysis in Section 6. Section 7 concludes.

2 Issues in constructing IV indexes

Gastineau (1977) proposed the use of an index to resolve the problem of multiple values of IVs from different options on the same underlying. An IV index calculated as a weighted average of the IVs from different option prices, would be the summary measure of underlying future volatility.

The first weighting schemes were suggested by Trippi (1977) and Schmalensee and Trippi (1978) which placed equal weights on all the IVs used in calculating the index. However, since the literature showed that the Black-Scholes model priced some options more accurately than others, schemes where the weights varied according to different factors were proposed. In following years, several researchers made significant progress in developing these concepts further (Galai, 1989; Cox and Rubinstein, 1985; Brenner and Galai, 1993; Whaley, 1993).

The maturation of knowledge in this field was signalled with the launch of an information product in 1993: an IV index based on trading in options on the S&P 100 index. This was called the CBOE VIX. A research literature rapidly demonstrated that VIX was useful in volatility forecasting, over and beyond the state of the art volatility models, since option prices harnessed the superior information set of traders.[1]

Given the importance of volatility indexes such as VIX in the global financial system, it is useful to explore the methodological issues in the construction of these indexes. The process of creating an optimal methodology for a volatility index involves two parts:

1. Identifying alternative weighting schemes based on available data about factors that directly influence the shape of the IV smile.
2. Choosing an optimal weighting scheme.

2.1 Factors influencing IV values

If the Black-Scholes model held exactly, all options should have the same implied volatility. However, an extensive literature has demonstrated that IV varies with moneyness, maturity, vega and liquidity. We discuss each of these in turn.

[1] Christensen and Prabhala (1998); Christensen *et al.* (2001) identified and corrected some of the data and methodological problems present in the early studies on this question. They conclude that IV is a more efficient forecast for future volatility than volatility calculated from historical returns. Latane and Rendleman (1976), Chiras and Manaster (1978), and Beckers (1981) find that IV performs better in capturing future volatility than standard deviations obtained from historical returns. Blair *et al.* (2001) find that volatility forecasts provided by the early CBOE volatility index are unbiased, and they outperform forecasts augmented with GARCH effects and high-frequency observations. Similar results were reported early on by Jorion (1995) for foreign exchange options.

Corrado and Miller (2005) examine the forecasting quality of three implied volatility indexes based on S&P 100, S&P 500 and Nasdaq 100. They find that the forecasting quality of the volatility index based on the S&P 100 and S&P 500 has improved since 1995, and that those based on the Nasdaq 100 provides better forecasts of future volatility.

Moneyness/Strike The first documented variation in IV was as a function of strikes or the moneyness. IV was consistently lower for lower values of the moneyness of the option. This variation came to be known as the *volatility smile*. Rubinstein (1994), Jackwerth and Rubinstein (1996), Dumas *et al.* (1998) showed that the pattern of the implied volatility of the S&P 500 index options changed from a smile to a sneer after the 1987 crash.

Maturity Prices of near month options show lower IV than far months. Heynen *et al.* (1994), Xu and Taylor (1994) and Campa and Chang (1995) show that implied volatilities are a function of time to expiration and thus exhibit a term structure.

Vega The derivative of the Black-Scholes price with respect to volatility is called vega. The vega can be shown to be consistently different for different values of the strike, as well as the maturity of the contract.[2] Thus the vega of an option naturally lent itself as an input to differentiating the IV of different options when calculating an IV index (Latane and Rendleman, 1976). Chiras and Manaster (1978) suggested weighting by volatility elasticity instead of vega.

Among other influential papers, Beckers (1981) and Whaley (1982) suggested minimizing $\sum_i w_i[C_i - BS_i(\hat{\sigma})]^2$ where C_i refers to market price and BS_i refers to Black Scholes price of option i and w_i could either be vega or equal weights.[3]

Liquidity A more recent literature has explored the impact of option liquidity on estimated IV. Brenner *et al.* (2001) show that there is a significant illiquidity premium between two sets of currency options, when one set is traded and the other is not. Bollen and Whaley (2004) documented an empirical link between the shape of the IV smile and the depth of the market on the buy and the sell side of options with different moneyness. They show that *net buying pressure* affects the shape of the IV smile in both the index as well as the single stock options markets. Further, they show that the shape of the IV smile is driven by different market forces for index options compared to single stock options.

[2] Vega is higher for options that are further away from the money since they have a lower extrinsic value and are less likely to change with changes in implied volatility. It is also higher for options with longer expiration in order to compensate for additional risk taken by the seller.

[3] This method thus allows the call prices to provide an implicit weighting scheme that yields an estimate of standard deviation which has least prediction error.

Models of asymmetric information have been used to provide theoretical underpinnings for the link between liquidity and option price. Nandi (1999) set up a model of asymmetric information linking the level and the shape of IV function to net order flow of options. The model shows that an increase in net options order flow increases the mispricing by the Black-Scholes model. Garleanu *et al.* (2009) formalise the findings in Bollen and Whaley (2004) by incorporating end-user demand in a model for options prices. Here they exploit the feature that end-users tend to hold long index options and short equity options to explain the relative expensiveness of index options. Another model to explicitly incorporate liquidity in the price of stock options was Cetin *et al.* (2006), who show market liquidity premium[4] as a significant part of the option price.

The empirical evidence has also linked IV to option liquidity. Etling and Miller (2000) explore the relationship between bid–ask spread as a liquidity proxy with moneyness of options and find that ATM options have the highest liquidity. Chou *et al.* (2009) explore how the IV function varies as a function of liquidity in both the spot and options market. They find that *order based* measures of liquidity (such as the bid–ask spread) better explain the variation in IV than *trade based* measures (such as traded volume). They also find that both spot and options markets liquidity matter for the variation in IV.

This evidence, about the various factors that influence IV, has led to many alternative approaches to constructing an IV index. The different weighting schemes are further discussed in Section 3. What is the *efficient* weighting scheme rests upon the performance of the forecast from each scheme against some benchmark volatility measure. We now examine the framework to carry out such a performance evaluation of different weighting schemes in the next section.

2.2 Performance evaluation

One of the reasons that there is no consensus on one best weighting scheme for a volatility index is the lack of an observable volatility. The time-series econometrics literature has extensive work on a framework to evaluate the performance of a volatility forecast even though volatility is not observed.

[4] The paper models the liquidity using a generic supply function where option price monotonically increases with size of order.

For example, these ideas have been used in testing the forecasts of volatility models such as GARCH, EWMA, etc. This framework has two broad approaches: one which delivers a relative measure of performance amongst a set of candidate models, and the other which delivers a measure of performance of each of the candidate models against a single benchmark.

These questions were revisited when intra-day data revealed a superior volatility proxy: realised volatility (RV). Once RV was observed, it became possible to measure how well IV forecasts the RV of the underlying asset over the life of an option. Most studies use a predictive regression of the IV estimate on future volatility where the goodness of prediction is measured through the coefficients of predictive regressions. The early studies by Day and Lewis (1988), Lamoureux and Lastrapes (1993) and Canina and Figlewski (1993) showed that IV is *not* a good predictor for future return volatilities.

The framework of *encompassing regressions* was then used to assess the predictability of IV estimates against other forecast variables. This framework addresses the relative importance of competing volatility forecasts and whether one volatility forecast subsumes all information contained in other volatility forecast(s). Within this approach, Poteshman (2000); Jiang and Tian (2005b); Corrado and Miller (2005) have found that IV estimates are biased, but efficient and informative relative to forecasts from other volatility estimates.

A recent study by Becker *et al.* (2007) used an approach that differs from the traditional forecast encompassing approach used in earlier studies and finds that the S&P 500 IV index does not contain any such incremental information relevant for forecasting volatility. Becker *et al.* (2009) compare the index against a combination of forecasts of S&P 500 volatility by using the Model Confidence Set *(MCS)* methodology and finds that a combination of forecasts outperforms individual model based forecasts and implied volatility.

In this paper, we use two steps to compare the performance of our volatility indexes:

1. Forecasting regressions following Christensen and Prabhala (1998) to test the information content of the volatility measures. We also run instrumental variable regressions to correct for potential errors-in-variable problems in implied volatility estimates as discussed by previous studies.[5]
2. The Model Confidence Set (MCS) methodology of Hansen *et al.* (2003). This addresses the problem of choosing the *best* forecasting model. It contains

[5] Christensen and Prabhala (1998), Jiang and Tian (2005b), Corrado and Miller (2005).

the best model with a given level of confidence. It may contain a number of models, which indicates they are of equal predictive ability. It has several advantages over other methods like superior predictive ability (SPA) test and the reality check (RC) test.[6]

The construction of the MCS test is an iterative procedure in that it requires a sequence of tests for equal predictive ability. The set of candidate models is trimmed by deleting models that are found to be inferior. The final surviving set of models in the MCS contain the optimal model with a given level of confidence and are not significantly different in terms of their forecast performance.[7]

The critical question that remains in this is still the choice of the benchmark measure for volatility which we discuss in Section 4.2.

3 Choices of IV indexes

In this section, we describe the different methods we use in order to calculate IV indexes. We start with a description of the two most widely computed volatility indexes by several exchanges across the world, namely, VXO and VIX.

3.1 VXO

This volatility index is calculated using prices of options on the S&P 100 index. The implied volatilities are calculated using the Black-Scholes model, and the VXO is an average of the IVs on eight near-the-money options, including options at the two nearest maturities.[8]

In 2003, VXO was criticised for using an option pricing model and being biased due to the trading day conversion. In addition there were two structural changes[9] in the US economy that reduced the usefulness of VXO as a measure of future volatility. These were:

1. S&P 500 options became the most actively traded index options.

[6]See Hansen *et al.* (2003).
[7]Hansen *et al.* (2003).
[8]See Whaley (1993) for details on construction of VXO.
[9]See Whaley (2009).

2. Earlier index calls and puts were equally important in investor trading strategies but in later years the market became dominated by portfolio insurers who bought out-of-the-money and at-the-money index puts for insurance purposes.

Such criticisms of VXO along with changes in the structure of the US options market led to a new approach to calculating the volatility index, VIX, based on the prices of options trading on the S&P 500.

3.2 VIX

In contrast to VXO, VIX has been derived from the concept of fair value of a volatility swap (Demeterfi *et al.*, 1999). Here, even though the variance is derived from market observable option prices and interest rates, the theoretical underpinning is rooted in the broader context of model-free implied variance of Dupire (1993) and Neuberger (1994). This concept was further developed by Carr and Madan (1998), Demeterfi *et al.* (1999) and Britten-Jones and Neuberger (2000). Jiang and Tian (2005a) establish that the variance measure under this framework is theoretically equivalent to the model-free implied variance formulated by Britten-Jones and Neuberger (2000).

The CBOE calculates and publishes a real time value of VIX.[10] which has been accepted as the market measure of volatility. In this paper, we do not directly analyse the VIX methodology. However, to the extent that the main argument of this paper is appropriate – that price information for illiquid option series is less informative – it should impact upon the VIX methodology also.

3.3 Volatility linked weights

The early literature (Latane and Rendleman, 1976; Chiras and Manaster, 1978), suggests two different weighting schemes based on vega and volatility elasticity weighting scheme to calculate the market IV index.

1. Vega weights are calculated as:

$$\sigma_{tj} = \frac{\sum_i w_{it,j}\sigma_{it}}{\sum_i w_{it,j}}$$

[10] See www.cboe.com/micro/vix/vixwhite.pdf for details on construction of VIX.

where $w_{it,j}$ is the Black-Scholes vega for the i^{th} option contract at time t. $j = 1, 2$ denotes the two nearest maturities.

2. Volatility elasticity weights are calculated as:

$$\sigma_{tj} = \frac{\sum_i w_{it,j} \frac{\sigma_{it,j}}{C_{it,j}} \sigma_{it,j}}{\sum_i w_{it,j} \frac{\sigma_{it,j}}{C_{it,j}}}$$

where $w_{it,j}$ is the Black-Scholes vega and $C_{it,j}$ is the price for the i^{th} option contract at time t , $j = 1, 2$ denotes the two nearest maturities.

The scheme that uses volatility elasticities puts more weight on out-of-the-money options (with low prices $C_{it,j}$) than the vega weights model.

3.4 Adjustment for rollover

The implied volatility estimates obtained for the two nearest maturity is linearly interpolated to obtain a 30 day estimate. Rollover to the next expiration occurs eight calendar days prior to the expiry of the nearby option. The interpolation scheme used is :

$$\text{VIX} = 100 \times \left[\sigma_{t1} \left(\frac{\mathrm{N}_{c2} - 30}{\mathrm{N}_{c2} - \mathrm{N}_{c1}} \right) + \sigma_{t2} \left(\frac{30 - \mathrm{N}_{c1}}{\mathrm{N}_{c2} - \mathrm{N}_{c1}} \right) \right]$$

where σ_{ti} are implied volatilities and N_{ci} is the number of calendar days to expiration. Here, $i = 1, 2$ for the near and next month respectively.

4 Measurement

We use data on the NSE-50 (Nifty) index options at the National Stock Exchange of India Ltd (NSE). The NSE is an extremely active exchange and is a high quality source of data on exchange-traded derivatives. NSE is the fifth largest derivative exchange in the world in terms of number of contracts traded (Table 1). It is also the third largest exchange in terms of number of contracts traded in equity index (Table 2).

Table 3 shows the average number of records of intra-day data for the Nifty index option contracts from March 2009 to April 2010. The large number of records suggests a highly active market.

Table 1 Global exchanges: number of contracts traded

Rank	Exchanges	Jan-Jun 2009	Jan-Jun 2010	% change
1	Korea Exchange	1,464,666,838	1,781,536,153	21.6%
2	CME Group	1,283,607,627	1,571,345,534	22.4%
3	Eurex	1,405,987,678	1,485,540,933	5.7%
4	NYSE Euronext	847,659,175	1,210,532,100	42.8%
5	National Stock Exchange of India	397,729,690	783,897,711	97.1%

Source: FIA, `http://www.futuresindustry.org/volume-.asp`

Table 2 Ranked by the number of contracts traded in equity index

Rank	Exchanges	Jan-Jun 2009	Jan-Jun 2010	% change
1	Kospi 200 Options, KRX	1,375,065,894	1,671,466,852	21.6%
2	Emini S&P 500 Futures, CME	308,764,146	299,603,623	3.0%
3	S&P CNX Nifty Options, NSE India	146,706,110	221,430,570	50.9%
4	SPDR S&P 500 ETF Options, CME	181,699,626	219,409,316	20.8%
5	DJ Euro Stoxx 50 Futures, Eurex	178,923,108	205,280,712	14.7 %

Source: FIA, `http://www.futuresindustry.org/volume-.asp`

Table 3 Records of intra-day data per month, with the Nifty index options market

Month	Avg no. of records
March 2009	2891142
April 2009	3596361
May 2009	2732869
June 2009	3409783
July 2009	4563929
August 2009	4724429
September 2009	4855215
October 2009	4790568
November 2009	6014413
December 2009	7768108
January 2010	7163127
February 2010	8429391
March 2010	7527610
April 2010	8767319

Source: NSE

The numerical values shown in the three tables (1, 2, 3) all show very high growth rates at NSE. Thus, while the NSE is an important exchange on the global scale, its importance is likely to go up in the future if these growth rates continue.

4.1 Price measurement

Even though we have both traded prices as well as mid–quote prices available at high frequency, we choose the mid–quote of the bid–ask orders of the options from the options limit order book as the benchmark input price in the Black-Scholes model to compute IV.

This is because of the relative illiquidity of the options market by way of trade updates compared to order updates. Very frequently, the next-month options market suffers from illiquidity in terms of trades, and there is no traded price that is observable. However, the order book has more liquidity in terms of order updates. Thus, there is information in the order book data that is not reflected in the traded prices. This makes it meaningful to use the mid–quote prices rather than traded prices to calculate IV for the volatility indexes since it reduces the effect of missing data.

The use of mid–quote prices has an automatic liquidity/illiquidity impact in the volatility index calculation – where the options prices are moving due to changes in the limit order book rather than a realised price from a transaction. This problem is prevalent in other emerging markets, and has implicitly driven an incorporation of liquidity considerations into volatility index calculations (Tzang *et al.*, 2010).

4.2 Measurement of realised volatility

Of the volatility indexes discussed in Section 3, we aim to compute the following four using the Nifty options data. These are: VXO, VVIX, EVIX and SVIX. We then plan to compare the dynamic behaviour of these different indexes with realised volatility (RV) as the benchmark measure of market volatility.

The theory of quadratic variation suggests that, under suitable conditions, RV is an unbiased and highly efficient estimator of volatility of returns (Andersen *et al.*, 2001). RV is computed as sum of intra-day squared returns. RV over [0,T] is defined as:

$$\mathrm{RV}_T = \Sigma_{i=1}^{n} r^2_{\frac{iT}{n}}$$

here $r_{\frac{iT}{n}}$ refers to index returns from time $(i-1)\frac{T}{n}$ to $i\frac{T}{n}$.

For the calculation of RV, we use data on Nifty index price which is available at within-one-second intervals from the trades and orders dataset. As a first step, we discretise this data at 10-minutes. This discretised data is then used to calculate daily market index volatility.

Earlier studies like Canina and Figlewski (1993) use overlapping samples to evaluate the performance of implied volatility estimates, while other studies like Christensen and Prabhala (1998), Jiang and Tian (2005b), Corrado and Miller (2005) use non-overlapping samples by using data at a lower frequency (monthly) in evaluating the performance of implied volatility estimates.

For our analysis, all volatility indexes are reduced to daily values (at the end of the trading day), by dividing them by the square root of the number of calendar days, 365. Since volatility indexes are ex–ante measures of the volatility, we adjust each days volatility index to the next period.

5 A volatility index that explicitly utilises liquidity in weights

The linkages between liquidity and implied volatilities presented in Section 2.1 appear to lead to a calculated VIX value which may be biased due to illiquidity and non–continuous strike prices. The literature has documented that across different underlyings, options on less liquid underlyings have a larger premium compared to those on more liquid underlyings. An extreme version of the difficulties caused by illiquidity is documented in Jiang and Tian (2005a), who found that the VIX constructed by the CBOE is flawed due to truncation errors that arise from the unavailability of option data for very low and very high strikes in practice.

We propose two elements of a strategy for confronting the problem of illiquidity. First, we utilise the mid–quote price rather than traded prices. This reduces noise. Secondly, we explicitly weight option IV by option liquidity, which we measure as the bid-ask spread available at that point in time in the limit order book for that option. These weights are calculated as follows:

$$\sigma_{tj} = \frac{\sum_i w_{it,j}\sigma_{it}}{\sum_i w_{it,j}}$$

where $w_{it,j} = 1/s_{it,j}$ and $s_{it,j}$ refers to the percentage spread defined as (ask-bid)/mid-price of option i at t, and $j = 1, 2$ stands for the two nearest maturities.

This strategy attaches greater weight to liquid products, where observed prices or quotes have reduced noise. The lack of availability of options prices traded at a wide range of strikes is known to magnify the truncation error of the CBOE VIX calculation methodology, and increase the bias of the VIX measure. Our method automatically adjusts for the lack of data by incorporating it in the value of the spread. If there is data missing on either side of the book, the spread would take a value of infinity, and the weight attributed to that option would be zero.

5.1 Stylised facts on the cross-sectional variation of option liquidity

The crucial issue that affects this research is the cross-sectional variation of option liquidity. Our empirical work is based on Indian data. This raises the concern that the results are an artifact of this emerging markets setting - perhaps one where liquidity is spotty, where arbitrage is weak, or one where liquidity risk is large.

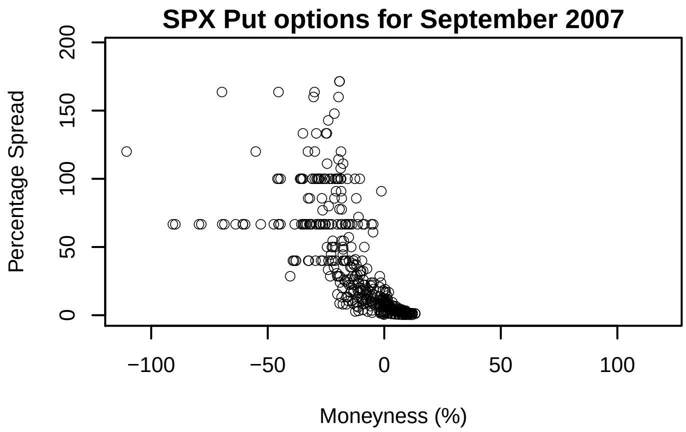
In order to evaluate this question, we plot bid-offer spreads on put options in the US (Figure 1) and in India (Figure 3). We also plot bid-offer spreads on call options in the US (Figure 2) and in India (Figure 4).

In both countries, we see high cross-sectional variation of option liquidity. If anything, option illiquidity is a smaller problem in India. Thus, our empirical results may be biased towards understating the gains from bringing liquidity considerations integrally into the construction of an implied volatility index.

Further, all four figures show that in the crisis period (September 2008), option illiquidity was a much bigger issue when compared with a tranquil period (September 2007). This suggests that the importance of this work would be enhanced under stressed market conditions.

Figure 1 Variation of put option spreads, US

These graphs show the relationship between percentage spread and moneyness for the the US market index options markets for the month of september 2007 (pre-crises) vs. september 2008 (crises). The first is the plot of the *put options market* on the S&P 500 at the Chicago Board Options Exchange (CBOE) with near month expiry for the month of september 2007. On the y-axis is the percentage spread (%) and on the x-axis is the moneyness of the option, calculated as (Strike - Current index level)/(Strike) and also expressed in %. Similarly, the second is the plot of the *put options market* for the market index at CBOE with near month expiry for the month of september 2008. The graphs show that put spreads worsened during the crises period in the US options market.



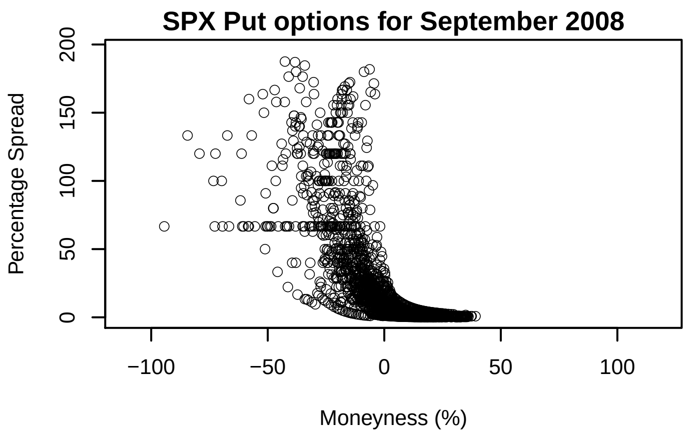


Figure 2 Variation of call option spreads, US

These graphs show the relationship between percentage spread and moneyness for the US market index options marketsfor the month of september 2007 (pre-crises) vs. september 2008 (crises). The first is the plot of the *call options market* on the S&P 500 at the Chicago Board Options Exchange (CBOE) with near month expiry for the month of september 2007. On the y-axis is the percentage spread (%) and on the x-axis is the moneyness of the option, calculated as (Current index level-strike)/(Strike) and also expressed in %. Similarly, the second is the plot of the *call options market* for the market index at CBOE with near month expiry for the month of september 2008. The graphs show that call spreads worsened during the crises period in the US options market.

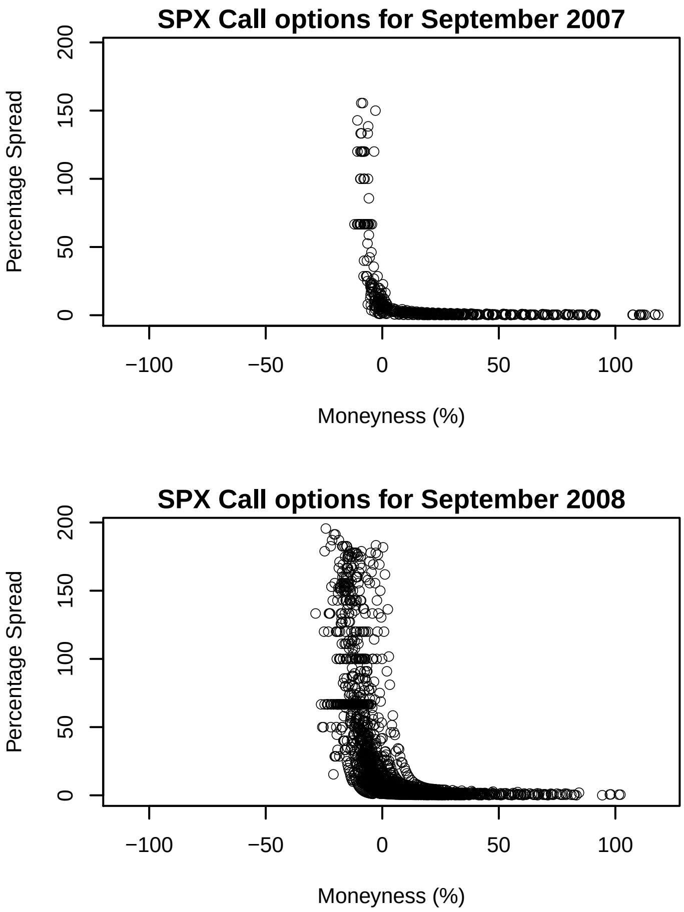
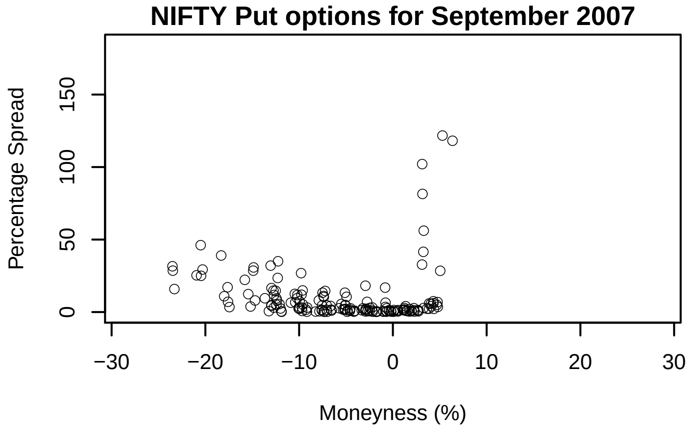


Figure 3 Variation of put option spreads, India

These graphs show the relationship between percentage spread and moneyness for the Indian market index options markets for the month of september 2007 (pre-crises) vs. september 2008 (crises). The first is the plot of the *put options market* on the NIFTY with near month expiry for the month of september 2007. On the y-axis is the percentage spread (%) and on the x-axis is the moneyness of the option, calculated as (Strike - Current index level)/(Strike) and also expressed in %. Similarly, the second is the plot of the *put options market* on the NIFTY with near month expiry for the month of september 2008. The graphs show that put spreads worsened during the crises period in the Indian options market.



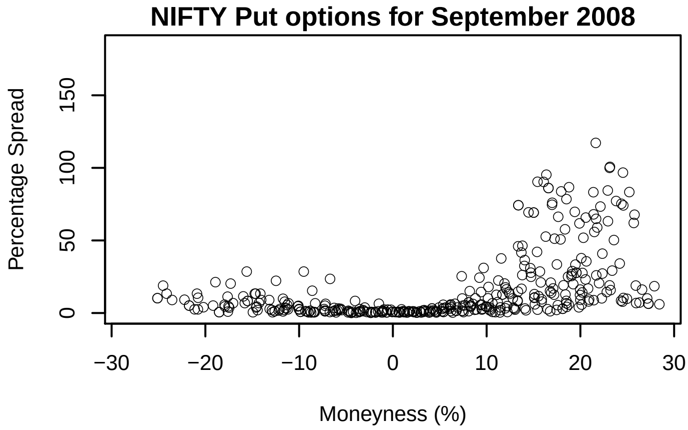
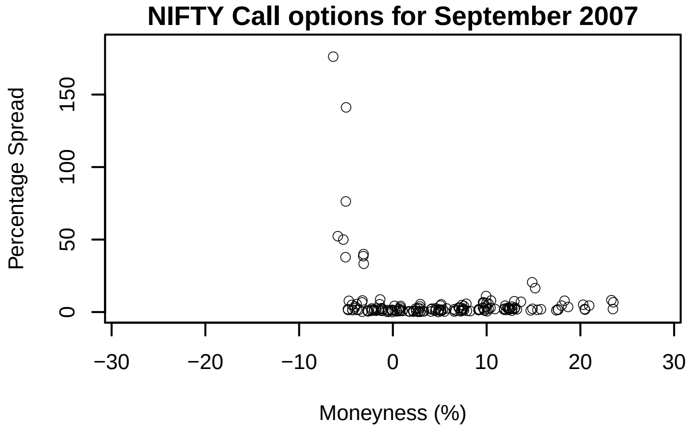
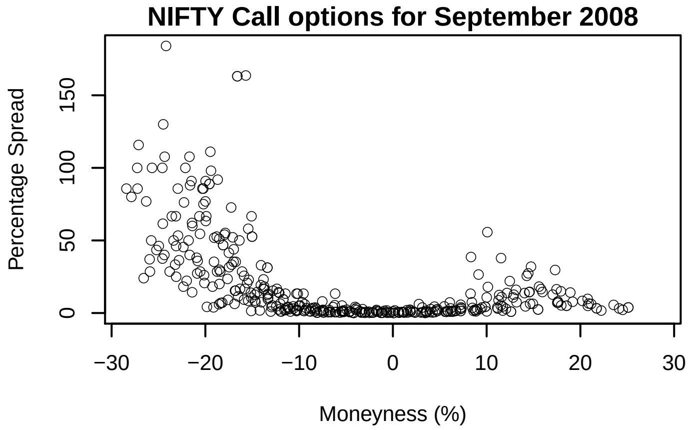


Figure 4 Variation of call option spreads, India

These graphs show the relationship between percentage spread and moneyness for the Indian market index options markets for the month of september 2007 (pre-crises) vs. september 2008 (crises). The first is the plot of the *call options market* on the NIFTY with near month expiry for the month of september 2007. On the y-axis is the percentage spread (%) and on the x-axis is the moneyness of the option, calculated as (Current index level-strike)/(Strike) and also expressed in %. Similarly, the second is the plot of the *call options market* on the NIFTY with near month expiry for the month of september 2008. The graphs show that call spreads worsened during the crises period in the Indian options market.





6 Empirical results

We have proposed three alternative weighting schemes in Section 3 through which IV is estimated. The first weighting scheme uses the Black-Scholes vega (VVIX), the second uses volatility elasticity (EVIX) and the third uses the bid offer spread in order to construct the weights (SVIX) for computing a volatility index.

The performance of these three volatility indexes, along with the old CBOE methodology (VXO) is compared against the benchmark of realised volatility (RV). We compare the performance of the volatility indexes in two ways:

1. Forecasting regressions that test the information content of the volatility indexes. We also run instrumental variable regressions to correct for potential errors-in-variables problems in implied volatility estimates.
2. The MCS methodology which allows comparison of multiple volatility forecasts and chooses the volatility forecast which is best in tracking RV.

Figure 5 shows how each volatility index tracks the RV. A common feature is that all the candidates appear to be an overestimate of volatility, which is measured as RV. One possible reason for the bias is that RV is computed as a sum of intra-day squared returns from opening of trading to the closing [11], and does not include the close to open volatility. This is unlike the assumption about the IV as a forecast of the volatility of returns which is calculated as price change from closing to closing of the day. For example, data on |R| compared to RV for the four quarters in Table 4 shows that volatility of closing-to-closing returns tends to be higher on average than the RV.

The graph of SVIX clearly indicates that it is best in tracking RV followed by VVIX. Differences between RV on day t and IV observed on the day $t - 1$ represent observed forecast errors.

Table 5 gives the summary statistics for each type of volatility index for both the raw and log values. They are all higher on average than the corresponding RV series. The IV values are thus likely biased forecast for RV. In addition, the reported skewness and kurtosis reveal that the log volatility index is more conformable with the normal distribution while the volatility index itself is not.

[11] See Section4.2.

Table 4 RV vs. |R|

The table presents the difference between $\bar{\text{RV}}$ and $|\bar{\text{R}}|$ for market index returns in four quarters of data. 2009Q3 is the quarter covering the period Oct, Nov, Dec 2009 while 2010Q2 includes Jul, Aug, Sep 2010. RV is computed as explained in Section 4.2 while |R| is computed as $|\text{R}_t| = |\ln(p_t/p_{t-1})|$ where p_t refers to closing price of the market index on day t.

Nifty	2009Q3	2009Q4	2010Q1	2010Q2
$\lvert\bar{\text{R}}\rvert$	15.21	14.31	15.05	12.07
$\bar{\text{RV}}$	15.23	11.57	11.00	8.25

Figure 5 Volatility indexes vs. RV

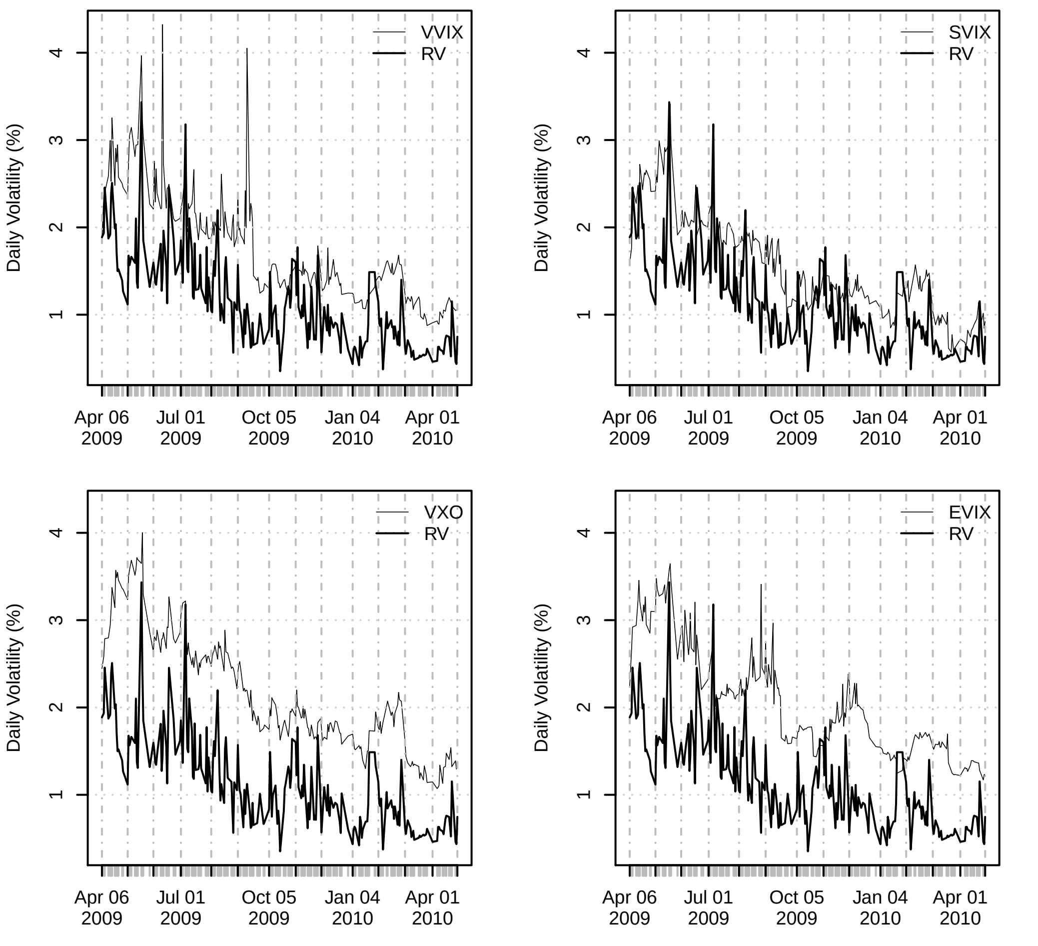


Table 5 Summary statistics of RV and IV

	Volatility					Log Volatility				
	RV	VXO	VVIX	EVIX	SVIX	LRV	LVXO	LVVIX	LEVIX	LSVIX
No. of obs.	216	216	216	216	216	216	216	216	216	216
Min	0.35	1.07	0.88	1.17	0.54	-1.04	0.07	-0.13	0.16	-0.62
Max	3.43	4.00	4.32	3.64	3.41	1.23	1.39	1.46	1.29	1.23
Mean	1.13	2.16	1.78	2.05	1.55	0.02	0.72	0.52	0.68	0.37
Kurtosis	4.85	2.58	4.26	2.68	3.04	2.36	2.24	2.40	2.15	2.65
Skewness	1.14	0.58	1.07	0.74	0.65	0.05	0.03	0.30	0.28	-0.17
Std Dev	0.52	0.66	0.63	0.61	0.56	0.45	0.30	0.34	0.28	0.36
Q1	0.70	1.67	1.31	1.57	1.15	-0.35	0.52	0.27	0.45	0.14
Q2	1.04	1.98	1.57	1.92	1.44	0.04	0.68	0.45	0.65	0.36
Q3	1.45	1.90	2.17	2.35	1.90	0.37	0.97	0.78	0.37	0.64

Table 6 Regression results

Note: For each regression, the t statistic is computed by following a robust procedure taking into account the heteroscedastic and autocorrelated error structure ((Newey and West, 1987)). The parenthesis below each coefficient reports the p-value.

Volatility Indexes	a_0	a_1	Adj.R^2	χ^2	DW
LVXO	-0.83	1.17	0.62	731.1	1.38
	(0.00)	(0.00)		(0.00)	
LVVIX	-0.50	1.01	0.57	249.1	1.23
	(0.00)	(0.00)		(0.00)	
LEVIX	-0.69	1.05	0.43	269.0	0.99
	(0.00)	(0.00)		(0.00)	
LSVIX	-0.33	0.95	0.59	153.5	1.39
	(0.00)	(0.00)		(0.00)	

6.1 Volatility forecast regressions

We run univariate regressions of log RV on each of the log volatility indexes separately to test several hypothesis associated with the information content of the volatility measures. Regressions are run using log volatility series to ensure that the probability density of the error term is close to the normal density. If the volatility forecast contains no information about the future volatility then the slope coefficient a_1 would be zero.

We consider

$$\mathrm{LRV}_t = a_0 + a_1 \mathrm{LVIX}_{i(t-1)} + \epsilon_t$$

Here LVIX_i belongs to the set of log values of VXO, VVIX, EVIX, SVIX.

Table 6 summarises the regression results. The slope coefficient is positive and significant at 1% for all volatility indexes indicating that all of them contain important information about future volatility. The slope coefficient is not significantly different from one for VVIX, EVIX and SVIX.

If a given volatility forecast is an unbiased estimator of future realised volatility, the slope coefficient a_1 should be one and the intercept a_0 should be zero. The null hypothesis of no bias is tested using the Newey-West covariance matrix. χ^2 statistics and p values are reported in Table 6. The null hypothesis is rejected at the 1% significance level in all cases with estimated coefficients a_1 ranging from 0.95 to 1.17.

This result is not surprising because summary statistics in Table 5 indicate that all volatility indexes are on average greater than the RV. The evidence is also consistent with the existing option pricing literature which documents that stochastic volatility is priced with a negative market price of risk (or equivalently a positive risk premium). The volatility implied from option prices is thus higher than their counterpart under the objective measure due to investor risk aversion (Jiang and Tian, 2005b).

The adjusted R^2 of VXO is slightly higher than SVIX while VVIX and EVIX have adjusted R^2 lower than SVIX. However, the difference of 3% is not sufficiently high to conclude that VXO has a better predictive ability than SVIX. The DW statistic is significantly different from 2 indicating that the residuals still reflect dependence across time points.

6.2 Instrumental variable regressions

The instrumental variable approach is adopted when there may be possible errors in explanatory variables. Many studies such as Christensen and Prabhala (1998), Jiang and Tian (2005b), Corrado and Miller (2005) have discussed the possible reasons that may result in the error-in-variable problem in implied volatility estimates which may further bias the slope coefficient in the univariate regressions discussed earlier. Following Christensen and Prabhala (1998), we apply a two-stage least squares regression to implement the instrumental variable estimation procedure. We use lagged IV as an instrument for IV. In the first stage, we regress each volatility index on the instrumental variable. In the second stage, RV is regressed on the fitted values obtained from the regression in the first stage.

Table 7 Instrumental variable regression results:

Dependent Variable: $\widehat{\text{LVIX}_{i(t-1)}}$					Dependent Variable: LRV_t					
1^{st}Stage	a_0	a_1	Adj.R^2	DW	2^{nd} Stage	b_0	b_1	Adj.R^2	χ^2	DW
LVXO_{t-2}	0.01 (0.29)	0.98 (0.00)	0.94	2.44	$\widehat{\text{LVXO}_{t-1}}$	-0.83 (0.00)	1.17 (0.00)	0.59	598.4 (0.00)	1.29
LVVIX_{t-2}	0.04 (0.02)	0.92 (0.00)	0.83	2.68	$\widehat{\text{LVVIX}_{t-1}}$	-0.52 (0.02)	1.04 (0.00)	0.51	201.8 (0.00)	1.22
LEVIX_{t-2}	0.03 (0.02)	0.95 (0.00)	0.89	2.63	$\widehat{\text{LEVIX}_{t-1}}$	-0.73 (0.00)	1.10 (0.00)	0.43	243.5 (0.00)	0.97
LSVIX_{t-2}	0.02 (0.02)	0.94 (0.00)	0.87	2.65	$\widehat{\text{LSVIX}_{t-1}}$	-0.34 (0.00)	0.97 (0.00)	0.54	147.3 (0.00)	1.29

We consider the following:

$$\widehat{\text{LVIX}_{i(t-1)}} = a_0 + a_1 \text{LVIX}_{i(t-2)}$$

$$\text{LRV}_t = b_0 + b_1 \widehat{\text{LVIX}_{i(t-1)}} + \epsilon_t$$

Table 7 summarises the results for the instrumental variable regressions. We find no material change in statistical inferences between instrumental variable and OLS regressions.

6.3 MCS results

Table 8 reports the rankings of all log volatility indexes based on mean square error (MSE), as well as the MCS results. The MSE errors are the first column of data in the table which shows that LSVIX has the smallest MSE and is thus the most accurate forecast of future volatility.

The remainder of the columns show the MCS results. Here, it turns out that the model with the largest range statistic T_r is VXO. The p-value in the first reduction is 0.019. As it is eliminated in the first round, this automatically determines that the MCS p-value for VXO is 0.019.

In the second round EVIX is eliminated with a p-value of 0.011. Since this p-value is smaller than the MCS p-value of model previously dropped hence the MCS p-value for EVIX is 0.019.

In the third round VVIX is eliminated with a p-value of 0.006. As this p-value is smaller than the MCS p-value of model(s) previously dropped the MCS

Table 8 Loss function rankings and MCS results

VIX	MSE	p_{T_r}	MCS(p_{T_r})	$p_{T_{SQ}}$	MCS($p_{T_{SQ}}$)
LVXO	0.392	0.019	0.019	0.000	0.000
LEVIX	0.304	0.011	0.019	0.000	0.000
LVVIX	0.201	0.006	0.019	0.006	0.006
LSVIX	**0.112**	-	**1.000**	-	**1.000**

p-value for VVIX is 0.019. The results remain the same when we look at the semi-quadratic statistic T_{SQ} rather than T_r.

The only volatility index that survives in the MCS is SVIX while all other volatility indexes are dropped at 5 % level of significance.

6.4 Sensitivity analysis

The main strategy of this paper has involved the bid/offer spread as a measure of option liquidity, and weights which vary inversely with the spread. There is a role for exploring alternatives to both these foundations of the research.

In recent work, Chaudhury (2011) propose two alternative measures of option liquidity:

Measure 1 s $= (ask - bid)/vol$

$$vol = S \times \sigma \times \sqrt{1/252}$$

here S refers to underlying asset price, σ refers to implied volatility of option.

Measure 2 s $= (ask - bid)/(\frac{\delta V}{\delta \sigma} \times \sigma)$

here V refers to the the mid-price of option and σ refers to implied volatility of option.

The analysis of this paper was repeated using both these measures. The volatility index computed using either Measure 1 or Measure 2 is inferior to our main work.[12]

Another direction of exploration is the variation of the weight by option spread. The main work of this paper has employed weights $w = 1/s$. This is

[12]Detailed results are available on request from the authors.

an ad-hoc specification lacking theoretical rationale. Hence, we also explore two alternative specifications:

$$w = \frac{1}{s^2}$$

and

$$w = \frac{1}{\sqrt{s}}$$

The former has weights that rapidly drop off, when the spread widens, and the latter has weights that drop off relatively slowly. Neither of these alternatives yielded an improvement when compared with the main work.[13]

7 Conclusion

The VXO and VIX are widely accepted volatility indexes and are computed by many exchanges across the world. However, options markets show substantial cross-sectional variation in liquidity. This cross-sectional variation is accentuated in crisis periods. Price information for illiquid options is less informative. The present strategies for construction of volatility indexes err in treating all price data as equally informative.

The contribution of our paper lies in isolating this issue, and proposing a volatility index where the option IV, which is computed using the midpoint quote, is weighted by the inverse of the bid-offer spread of the option.

Our work falls under the larger theme of bringing microstructure considerations more integrally into the utilisation of information from financial markets Shah and Thomas (1998). Some markets which are highly liquid in industrial countries may be relatively illiquid in emerging markets. While some traded products (e.g. ATM options) might be highly liquid, other traded products might be illiquid. While some markets may be ordinarily highly liquid in ordinary times (e.g. the US TIPS market), they may become illiquid under stressed conditions. This microstructure perspective can be useful with many applications of financial market data.

Our results indicate that the liquidity weighted volatility index (SVIX) outperforms other volatility indexes when compared against future realised volatility. In an ideal world, if all option series are identically liquid, then the SVIX would yield an answer which is no different from the conventional scheme:

[13] Detailed results are available on request from the authors.

our proposed scheme does no harm at times when all options are highly liquid, but it improves matters when cross-sectional variation in option liquidity occurs. This improved methodology is thus potentially useful in improving measurement of implied volatility at option exchanges worldwide.

In this paper, the simplest strategy – weighting by the inverse spread – proved to yield a volatility index that was superior to traditional methods. More generally, a superior volatility index might involve utilising information in both vega and in the bid-offer spread, and can be an interesting avenue for future research.

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